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STATISTICAL QUALITY CONTROL OF ENGINEERED EMBANKMENTS

by

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This report is an introduction to practical concepts, definitions, and techniques of statistical quality control for geotechnical engineering, with particular attention to compaction control of engineered embankments. The report briefly covers applicable fundamentals of statistical theory, followed by more specific coverage of (a) sampling theory, (b) quality control charts, and (c) acceptance sampling. Construction data from new data projects are used to illustrate the methods discussed. The report is intended for practical use and presumes no prior familiarity with statistical theory.

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PREFACE

This report, prepared by Gregory B. Baecher of NEXUS Associates, Wayland, Massachusetts, with assistance from D. DeGroot, C. Erikson, and A. Pais, under Contract No. DACW39-83-M-0067, provides details for the statistical analysis of geotechnical engineering aspects of new dam projects. It was part of work done by the US Army Engineer Waterways Experiment Station (WES) in the US Army Civil Works Investigation Study sponsored by the Office, Chief of Engineers, US Army. This study was conducted during the period October 1983 to September 1985 under CWIS Work Unit No. Civis 32221, entitled Probabilistic Methods in Soil Mechanics. Mr. Richard Davidson was the OCE Technical Monitor.

The report presents an introduction to statistical quality control as applied to the construction inspection of engineered embankments. It is intended to be introduction to potential users who have little or no background in statistics. Examples in the report are drawn from actual construction records of dam projects, and IBM-compatible microcomputer software supporting this report has been developed under separate funding. Two other instructional reports were prepared under the same contract, "Statistical Analysis of Geotechnical Data," and "Error Analysis for Geotechnical Engineering," in addition to a final report.

Ms. Mary Ellen Hynes-Griffin, Earthquake Engineering and Geophysics Division (EEGD), Geotechnical Laboratory (GL), WES, was the Contracting Officer's Representative and WES Principal Investigator for CWIS Work Unit 32221. General supervision was provided by Dr. A. G. Franklin, Chief, EEGD, and Dr. W. F. Marcuson III, Chief, GL.

COL Dwayne G. Lee, CE, was Commander and Director of WES during the publication of this report. Dr. Robert W. Whalin was Technical Director.
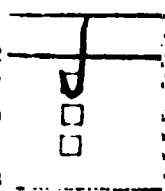
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PART I: INTRODUCTION

Background

Concern with quality and the control of manufacturing or construction

processes to assure quality underlie modern engineering and production.

Indeed, quality control is as old as engineering itself. On the other hand,

statistical quality control is relatively recent. In the United States,

statistical quality control first came into its own with the wartime production

effort of 1939-1945, the main impetus of this push having been Army Ordinance

and the War Production Board. The military influence has been important to the

introduction of statistical quality control to American industry.

Quality control in construction has characteristics which are both similar

to and different from quality control in manufacturing. The control of quality

in dam projects, especially concerning the placement and compaction of

engineered embankments, is critical to the safety and performance of the entire

project. Consequently, a well planned inspection program is considered

essential on any moderately large project. Current CE guidance for quality

control of engineered embankments is contained in EM 1110-2-1911, dated 17

January 1977. It is not statistically based, but is experience based.

Purpose

The purpose of this report is to provide potential users of statistical

quality control of engineered fills with an introduction to practical concepts,

definitions, and techniques. The report presents simple techniques which are

intended for use by readers having limited familiarity with statistical theory.

The report does not attempt to survey the literature of statistical quality control, but concentrates on a few chosen techniques that fill the needs of geotechnical engineering practice.

General Description of Statistical Quality Control

The placement of compacted fills, like any manufacturing or construction process, varies with time. The physical properties of soils being placed varies in moisture content, gradation, plasticity and other ways; and the process of placing soils varies, for example, in lift thickness, compactive effort, and climatic conditions. These variations cause physical properties of a resulting fill to differ from one point to another. A field inspection program intends to ensure that--to an acceptible level of confidence--the completed fill conforms to specified standards and thus will perform its function acceptably.

Ideally, an inspection program could non-destructively screen all soil placed in a fill and reject those materials with engineering properties not conforming to specified standards of strength, deformability or permeability. Such a program would guarantee perfection by detecting all parts of a fill which were flawed. Unfortunately, cost and the lack of reliable testing technology precludes this ideal program. Instead, typical inspection programs consist of limited numbers of small-scale tests spread thinly throughout a fill. The properties measured by most of these tests--for example, moisture content and dry density--are merely surrogates for the engineering properties of actual interest, although some engineering properties are also measured

Statistical quality control uses simple probalility theory to develop inspection sampling plans. These plans make efficient use of resources, and can be related to a quantitative confidence in the quality of a finished product.

Organization of This Report

This report is organized in six parts. After the introduction, Part II summarizes fundamentals of probability and statistics which are necessary for later presentations. Part III presents basic concepts of statistical quality control including sampling theory. Part IV briefly reviews field control of compaction operations. Part V presents quality control chart techniques. Finally, Part VI discusses the design (i.e., planning) of sampling schemes for field use. Following each chapter are tables and figures, and plates presenting example calculations.

PART II: FUNDAMENTALS

This section briefly reviews mathematical concepts underlying statistical quality control.

Probability Theory

Probability theory is a branch of pure mathematics. It is logical and internally consistent in the sense that all the mathematics of probability theory can be derived from a small set of axioms. In essence, the axioms specify properties that "probability" must have, for example probability is a real number between zero and one. Yet, nowehere do the axioms say what the concept of probability means. As a result many interpretations of what probability means are in common use.

Frequency

In statistical quality control, probability is usually interpreted to be the frequency of occurrence of some event in a long series of similar trials. A trial is an individual occurence producing an outcome of some sort. For example, each individual lift of soil placed in a compacted embankment might be considered a trial. The frequency of soils, having low moisture content among these lifts (i.e., among the trials) would be the probability of soil with low moisture content.

Subjective Probability

An alternative interpretation, common in geotechnical engineering, holds that probability is a rational degree of belief. The probability that an

important solution cavity exists in a limestone dam abutment is typical of geotechnical problems which cannot be easily approached using the frequency definition of probability. Such probabilities have to do with one-time events, past experience, and amounts of information. They are personal and subjective. They are not related to frequencies, actual or conceptual.

In this report the frequency definition of probability is used, for it is appropriate to quality control problems.

Randomness

A key concept of the frequency approach to probability is randomness. There are two places where the concept of randomness is important. One is the description of a construction process as operating in a random manner; the other is the design of a random sampling plan.

A process is operating in a random manner when any part of the output may be viewed as typical of the output as a whole. That is, when perturbations show no discernable pattern. Usually it is not possible to demonstrate that a process is operating randomly; rather, it is only possible to do the reverse, to demonstrate that a process is not random. This is done by showing that the output of the process does in fact have a pattern to it.

A process that is operating in a random manner has elements or events with definite probabilites of occurrence. These probabilities may not be known, or may be known only to the extent that data are available from which to draw estimates. Because the elements or events have associated probabilities, statistical theory and methods can be used to characterize them.

The other place randomness is important is in the design of random sampling plans. A random sampling plan is one in which sampled elements are chosen with definite probabilities, but without a predictable pattern. In large samples the relative frequency with which elements are sampled should approach those probabilities; however, the collection of elements which make up any specific sample reflect a chance distribution.

Conditional Probability and Independence

In quality control, probability is commonly defined as the relative frequency with which a certain event occurs in a long series of similar trials. For example, if there are N elements in a large set, of which n_a share a common property A, then the probability of an element within the set having property A is,

$$P(A) = \frac{n_a}{N} ,$$

-1-

If some of the elements also share a common property B, and if the number of these elements is n_b, then the probability of property B within the set is,

$$P(B) = \frac{n_b}{N} .$$

-2-

Consider now that some elements in the large set possess both property A and property B. Let the number of such elements be n_{ab}. Graphically, the number of elements possessing property A, property B, or both can be depicted

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as in Fig. 1. If we consider only those n_b elements having property B, the fraction of these also having property A is n_{ab}/n_b. This relative frequency is called the conditional probability of an element having property A, given that it is known to have property B. The conditional probability is denoted,

$$P(A|B) = \frac{n_{ab}}{n_b} \qquad\qquad -3-$$

By analogy, the conditional probability of property B given property A would be,

$$P(B|A) = \frac{n_{ab}}{n_a} \qquad\qquad -4-$$

The event that an element in the population possesses property A is said to be independent of the event that the element possesses property B when the probability of A is unchanged by knowing that an element possesses property B. Mathematically, A is independent of B if

$$P(A|B) = P(A) \qquad\qquad -5-$$

Knowing that the element possesses property B in no way influences the probability that it also possess property A. If the event that the element possesses property A is also independent of the event that it possesses property B, then properties A and B are said to be mutually independent.

Multiplication Theorem

Two theorems of probability theory are basic and often encountered in statistical quality control. These have to do with the relationships among the probabilities of distinct events. The first is the multiplication theorem.

The multiplication theorem states that the probability of two mutually independent events occuring simultaneously is the product of their individual probabilities. If elements possessing properties A and B within some large set are mutually exclusive events, then the probability of an element possessing both property A and property B is,

$$P(A \text{ and } B) = P(A) \ P(B) \ . \qquad\qquad -6-$$

If A and B are not mutually independent, the more general form of the multiplication theory states that the probability of them occuring simultaneously depends on the conditional probabilities.

$$P(A \text{ and } B) = P(A) \ P(B|A)$$
$$= P(B) \ P(A|B) \ . \qquad\qquad -7-$$

Addition Theorem

The addition theorem states that the probability of either one or the other of two events A and B occuring equals the sum of the individual probabilities of their occurrence, minus the probability that they occur together.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \ . \qquad\qquad -8-$$

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In Fig. 1, if area is taken to represent probability, the addition theorem can be considered simply a statement of geometry. The area contained by the combination of events A and B equals the sum of their individual areas, less one times the area of their overlap (i.e., P(A and B)) which would otherwise be double counted.

Frequency Distributions

The variability of data on production output, soil properties, or other variables is conv niently summarized in a frequency distribution, the fundamental tool used by statisticians.

Discrete and Continuous Variables

Fig. 2 shows the variability of standard penetration test blow counts measured in 40 borings in a silty sand deposit at a dam site. Blow counts can only assume interger values, and therefore are said to be discrete variables. Fig. 3 shows variability of water content measured in R-tests on 73 specimens of a compacted clay. These strength data may assume any real number value within a broad range, and are therefore said to be continuous variables. Quality control in geotechnical engineering must deal with both discrete and continuous variables, and many methods of statistical quality control apply to each in a similar way.

Histograms and Frequency Distributions

A convenient way to graphically represent scattered data is in a histogram. A histogram graphs the number of measurements falling within specific intervals of value as a vertical bar. Fig. 2 shows a histogram of SPT

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data. For obvious reasons, a histogram is sometimes called a bar chart. The height of the bar above each interval shows the number of measured values within the interval, and the sum of the heights of the bars equals the total number of measurements. Fig. 3 shows a histogram of R-test data.

The histogram of Fig. 3 divides the data into intervals of 1%. The choice of intervals is arbitrary, but the intervals should be of uniform width and have convenient end points. If too many intervals are chosen the general picture of relative frequencies will not be obtained, while conversely, if too few intervals are chosen the general picture will be blurred. A common rule-of-thumb is to use about 10 intervals. More detailed discussion is presented in the report "Data analysis for geotechnical engineering" (January 1986).

A frequency distribution is constructed from a histogram by dividing each vertical bar by the total number of measurements. This gives the relative frequency of observed value in each interval as a decimal fraction. The sum of the heights of the bars in a frequency distribution is 1.0. Fig. 4 shows the frequency distribution (right had side scale) corresponding to the histogram of Fig. 2. Fig. 5 shows the frequency distribution corresponding to the histogram of Fig. 3.

Cummulative Distribution

A cummulative distribution of discrete or continuous data is constructed by summing relative frequencies starting at the lower-value end of the data and proceeding toward the upper value end. The cummulative distribution

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denoted F(x) gives the fraction of measurements less than or equal to a
particular value,

$$F(x) = \text{fraction of measurements} \leqslant x. \qquad -9-$$

Cummulative frequencies for the data of Figs. 2 and 3 are shown in Figs. 6
and 7. The cummulative distribution has the properties that,

$$
\begin{aligned}
\text{For} \quad x &= \text{lower limit (or } -\infty) \quad \rightarrow \quad F(x) = 0 \quad ; \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -10- \\
\text{For} \quad x &= \text{upper limit (or } +\infty) \quad \rightarrow \quad F(x) = 1.0 \quad .
\end{aligned}
$$

For discrete data the cummulative distribution is a step function increasing to
the right. For continuous data the cummulative distribution is typically a
smooth S-shaped curve.

Importance of Frequency Distributions

Frequency distributions give a summary view of the variation in a set of
data. The shape of the distribution suggests whether the data have any central
tendency, and if so, where along the x-axis the data are concentrated. The
width of the distribution indicates the dispersion or scale of variation of the
data.

Some frequency distributions have one point of concentration and are
thus called unimodal. Others have more than one and are called multimodal.
Usually, soils data are unimodal. Multimodal distributions may indicate a
mixture of data from different soil types or different construction procedures,
that is, nonhomogeneous data.

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The frequency distribution also shows whether the variation in data is symmetric or asymmetric, that is, whether high and low variations are evenly balanced. For data that are asymmetrically distributed, large variation from the central tendency of the data set are more frequent on one side of the center than on the other. This is illustrated in Fig. 8.

Summary Statistics

Frequency distributions are convenient representations of data for visual inspection, but often numerical measures of distribution characteristics are useful for calculation or for setting standards. Numerical measures are essential for developing quality control criteria and quality control charts. The most important numerical measures pertain to the central tendency of data and to dispersion.

The term "statistic" refers to any mathematical function of a set of measured data. For example, given the measurements $x_1, \ldots, x_u$, any function $y = T(x_1, \ldots, x_u)$ is said to be a statistic of the data. The arithmetical average is such a function, the largest value x_{max} or the smallest value x_{min} is such a function, and so on. Any of these ways of summarizing the data would be called a statistic. Obviously, there are an infinite number of statistics which could be calculated from any set of data, but the most useful have either to do with the central tendency of the data along the x-axis or to the dispersion of the data.

Central Tendency

The most common measures of central tendency are the mean, median, and mode. The mean is the arithmetic average of a set of data. The median is the

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value for which half the observations are smaller and half larger. The mode is the most frequent value (Table 1).

The _mean_ of a set of n data $\underline{x} = \{x_1, \ldots, x_n\}$, denoted m_x, is defined as the arithmetical average,

$$m_x = \frac{1}{n} \sum_{i=1}^{n} x_i \qquad \text{-11-}$$

The mean is the center of gravity of the frequency distribution along the x-axis, as shown in Fig. 9.

The _median_ of the set of data $\underline{x} = \{x_1, \ldots, x_n\}$, denoted $x_{0.5}$, is the value of x_n which half the data are less than and half more than. The cummulative distribution evaluated at the median is 0.5,

$$F(x_{0.5}) = 0.5 \qquad \text{-12-}$$

The median is the midpoint of the data, when listed in increasing or decreasing order. Common practice in the case of an even number of data is to define the median as half way between the two middle data, that is, those of rank $(n/2)$ and $(n/2 + 1)$.

The _mode_ of the set of data $\underline{x} = \{x_1, \ldots, x_n\}$, denoted x_0, is the most often observed value. This is the value of x having the highest ordinate on the frequency distribution.

Dispersion

The most common measures of dispersion are the standard deviation, range, and inner quartiles.

The <u>Standard deviation</u> of a set of data $\underline{x} = \{x_1, \ldots, x_n\}$, denoted s_x, is defined as the root mean square variability of the data,

$$s_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - m_x)^2} \qquad -13-$$

in which m_x = the mean of the data. The denominator (n-1) rather than (n) is used to correct a statistical bias. In estimating the standard deviation from data, the mean is usually also unknown. Thus, the mean must be estimated from the same data as the standard deviation. This causes the average squared variability about m_x to be smaller than it should be. On average, it is smaller by a factor (n-1)/n. Correcting for this error gives Eqn. 13.

The <u>coefficient of variation</u> of a set of data is the standard deviation divided by the mean,

$$\Omega_x = s_x/m_x \qquad -14-$$

The coefficent of variation is used to express relative dispersion.

The <u>variance</u> of a set of data, denoted V_x, is the square of the standard deviation,

$$V_x = s_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - m_x)^2 \ . \qquad\qquad -15-$$

In many statistical calculations the variance is a more convenient term than the standard deviation, and is thus widely encountered in statistical quality control. The variance is the moment of inertia of the frequency distribution about m_x.

The range of a set of data, denoted r, is the difference between the largest and smallest values,

$$r_x = x_{max} - x_{min} \qquad . \qquad\qquad -16-$$

The range has poor statistical properties in that it is sensitive to extreme values in a data set, however, it is easily evaluated and therefore often useful.

The inner quartiles of a set of data, denoted $x_{0.25}$ and $x_{0.75}$, are the data values for which one-quarter of the data are smaller and one-quarter larger, respectively. The quartiles may be found from the cummulative distribution as

$$F(x_{0.25}) = 0.25 \qquad\qquad -17a-$$
$$F(x_{0.75}) = 0.75 . \qquad\qquad -17b-$$

The interquartile range, denoted $r_{0.5}$,

$$r_{0.5} = (x_{0.75} - x_{0.25}) \qquad\qquad -18-$$

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is less influenced by extreme values than is the range itself, but it is correspondingly more troublesome to compute. Various summary statistics applied to the R-test data of Fig. 3 are evaluated in Plate 1.

Association Among Uncertain Variables

When dealing with two or more soil properties the uncertainties in estimates may be associated with one another. That is, the uncertainty in one property estimate may not be independent of the uncertainty in the other estimate. Consider the problem of estimating 'cohesion' and 'friction' parameters of a Mohr-Coulomb strength envelope. If the slope of the envelope to the Mohr circles is mistakenly estimated too steeply, then for the line to fit the data the intercept will be too low. The reverse is true if the slope is estimated too flat. Thus, uncertainties about the slope and about the intercept are not independent, they are associated with one another.

The correlation coefficient for paired data $\underline{x}, \underline{y} = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ is denoted ρ_{xy}, and defined as,

$$\rho_{xy} = \frac{1}{n-2} \Sigma \left(\frac{x_i - m_x}{s_x}\right) \left(\frac{y_i - m_y}{s_y}\right) \qquad -19-$$

In effect, the correlation coefficient is equivalent to a normalized product moment of inertia in solid mechanics. It expresses the degree to which two parameters vary together. The correlation coefficient is non-dimensional because deviations of x and y are measured in the same units as their

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respective means.   The value of ρ_{xy} may vary from +1 to -1. $\rho_{x,y}=+1$ implies a strict linear relation with a positive slope; $\rho_{x,y}=-1$ implies a strict linear relation with a negative slope; $\rho_{x,y}=0$ implies no association at all (i.e., independence).

Quick Estimates

Often one wants quick, approximate estimates of means, standard deviations, or correlation coefficients from limited numbers of data.   Some shortcut techniques are available for this purpose.   These provide economies of time and effort while causing sometimes only minor losses of accuracy or precision.

Shortcuts for Estimating the Mean

Rather than using Equation 11, a quick and often good estimate of the mean can be obtained from the median.   The median is the middle value of a data set. For example, if, say, five data are listed in ascending order x_1, x_2, x_3, x_4, x_5, the median is x_3.   For an even number of data, say n=6, the difference between the two middle data is halved to give the median, that is $(x_3+x_4)/2$. For data scatter which is symmetric about its central value and for small numbers of data, the sample median is a good estimate of the true mean.   On the other hand, if the data scatter is asymmetric--for example, if there are many small values and a few large values--the sample median is not such a good estimator of the mean.

A second shortcut for estimating the mean is taking one-half the sum of the largest and smallest measured values, $(1/2)(x_{max} + x_{min})$.   This estimator

is sensitive to the extreme values in a set of measurements, and thus fluctuates considerably. It is not a good shortcut estimator and should only be used with caution.

Shortcuts for Estimating the Standard Deviation

Rather than using Equation 13, a quicker estimate of the standard deviation from small numbers of tests can be made from the sample range $r_x = (x_{max} - x_{min})$. The range is the span of data from largest to smallest. Like the standard deviation, the range is a measure of dispersion in a set of data. However, the relationship between the standard deviation and the sample range, on average, depends on how many tests are made. To obtain a best estimate of s_x from the range of data r_x a multiplier N_n is used which depends on sample size (Table 2). The best estimate of the standard deviation is $s_x \approx N_n r_x$ (see Plate 2).

As for the sample median, the range is a good estimator of the standard deviation for small n and symmetric data scatter. Even for modest n it remains fairly good. However, for asymmetric data scatter the range, which is strongly affected by outliers, is not a good estimator of the standard deviation. Fortunately, with the notable exception of hydraulic parameters such as permeability, most geotechnical data display symmetric scatter. In the case of hydraulic data a logarithmic transformation (Lee, et al., 1983) usually makes the data scatter symmetric, and again the median and range become convenient estimators.

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Shortcuts for Estimating the Correlation Coefficient

Calculation of correlation coefficients by Eqn. 19 can be time consuming and tedious. A simple and quick approximation is obtained graphically from the shape of the scatter plot of y vs. x. The method works well whenever the outline of the scatter plot is approximately ellipical, and works even with small numbers of observations. Using Chatillon's (1984) term and prodedure, this is called the balloon method:

STEP 1: Plot a scatter diagram of y vs. x.

STEP 2: Draw an ellipse (balloon) surrounding all or most of the points on the plot.

STEP 3: Measure the vertical height of the ellipse at its center, h, and the vertical height of the ellipse at its extremes, H.

STEP 4: Approximate the correlation coefficient as $r \simeq \sqrt{1 - (h/H)}$.

An example of the method is shown in Fig. 10. The balloon method gives a correlation coefficient of 0.81, whereas the correlation coefficient calculated by Eqn. 19 is 0.83. Empirically, the method works well for r>0.5.

Shilling (1984) suggests a similar balloon method for approximately estimating the correlation coefficient:

STEP 1: Plot a scatter diagram of $(y-m_y)/s_x$ vs. $(x-m_x)/s_y$.

STEP 2: Draw an ellipse surrounding all or most of the points on the plot.

STEP 3: Measure the length of the principal axis of the ellipse having positive slope, D, and the length of the principal axis of the ellipse having negative slope, d.

STEP 4: Approximate the correlation coefficient as $r (D^2 - d^2)/(D^2 + d^2)$.

This methods works about as well as Chatillon's. For the data of Fig. 10 Shilling's method gives r   0.80.

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Probability Distribution

For many problems in statistical quality control it is convenient to approximate the empirical frequency distribution for some category of data by a mathematical function. Surprisingly, a comparatively small set of mathematical functions can be used to fit a broad range of frequency distributions encountered in the field. By far the most important of these is the Normal or bell-shaped distribution. Among other useful distributions are the log Normal, Exponential, and 4-parameter Beta distribution, although many others exist. The Normal distribution is discussed here, while parallel properties of the other forms are given in Table 3.

The Normal distribution is represented by the equation

$$f(x) = \frac{1}{\sqrt{2\pi}\, s_x}\, e^{-\frac{1}{2}\left(\frac{x - m_x}{s_x}\right)^2} \qquad\qquad -20-$$

in which m_x = the mean of x and s_x = the standard deviation. The distribution is unimodal at m_x and symmetric (Fig. 11). The cummulative distribution of x using the Normal equation is found from the area under the frequency distribution up to x,

$$F(x) = \int_{-\infty}^{x} f(x)\, dx. \qquad\qquad -21-$$

The normal distribution is defined for $-\infty \leqslant x \leqslant +\infty$, but the area under the

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distribution beyond 3 to 4 standard deviations from the mean is neglible.

The area under the Normal distribution expressed as a function of the standardized variable

$$z = \frac{x - m_x}{s_x}$$

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and calculated by Eqn. 21 are given in Table 4. Benjamin and Cornell (1970) give examples. Numerically, these areas can be approximated by the series expansion (Abramowitz and Segun, 1964),

$$F(z) = f_N(x) \left(b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 \right) + \varepsilon$$

-23-

in which,

$$
\begin{aligned}
b_1 &= 0.319381530 & t &= (1+px)^{-1} \\
b_2 &= -0.356563782 & p &= 0.2316419 \\
b_3 &= 1.781477937 & |\varepsilon| &< 7.5 \times 10^{-8} \\
b_4 &= -1.821255978 \\
b_5 &= 1.330274429
\end{aligned}
$$

-24-

and $f_N(x)$ is the ordinate of the Normal distribution function evaluated at x. The series expansion is generally more convenient than Table 4 for use with computers.

If a construction process is operating in a random manner, and if good estimates of the mean and standard deviation are available, and if the frequency of data are observed to be well modelled by a representable distribution, then forecasts can be confidently made about the future performance of

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that process. This is the basis for statistical quality control. For example, if the process is observed to produce Normally distributed output, then a chart such as Fig. 12 can be constructed which shows the process mean and envelopes $\pm 3s_x$ about the mean. As long as the process continues to operate in a random manner, and the mean, standard deviation, and frequency distributions remain unchanged, then a confident forecast can be made that 99.7% of the output measurements to be made in the future will lie within the $\pm 3s_x$ bound (Figure 12). This forecast of 99.7% comes from Table 4. Such forecasts are considered in greater detail in Parts V and VI.

Table 1

Summary Measures of Frequency Distributions

Measure	Symbol	Formula	Comments
Central Tendency			
Mean	m_x	$1/n \sum x_i$	center of gravity
Median	$x_{0.5}$	$F(x_{0.5})=0.5$	middle value
Mode	x_0	$x_0 = \max\limits_{i} f(x_i)$	most frequent value
Dispersion			
Standard Deviation	s_x	$\sqrt{\dfrac{1}{n-1} \sum (x_i - m_x)^2}$	root mean square variation
Variance	V_x	s_x^2	moment of inertia about m_x
Range	r_x	$x_{max} - x_{min}$	
Interquartile Range	$r_{0.5}$	$x_{0.75} - x_{0.25}$	

Table 2

Ratio of average range to standard deviation for samples from a
Normal frequency distribution.

----

n	Multiplier N_n	n	Multiplier N_n
2	0.886	12	0.815
3	0.591	13	0.300
4	0.486	14	0.294
5	0.430	15	0.288
6	0.395	16	0.283
7	0.370	17	0.279
8	0.351	18	0.275
9	0.337	19	0.271
10	0.325	20	0.268
11	0.315		

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from Snedecor and Cochran (1980)

Table 3

Common Probability Distributions
(after Lee, et al (1983)
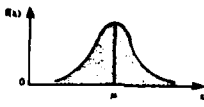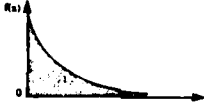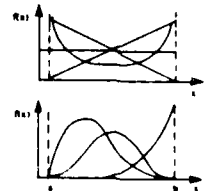
Type	Formula	Shape	Comments
Uniform	$f(x) = \dfrac{1}{(b - a)}$ for $a \leq x \leq b$		*Mean* $= \frac{1}{2}a + b)$; Variance $= \frac{1}{12}(b - a)^2$ Used when no reason to give other than equal likelihoods to possible values of x.
Normal	$f(x) = \dfrac{\exp\left[-\frac{1}{2}\left(\frac{x - \mu}{\sigma}\right)^2\right]}{\sigma\sqrt{2\pi}}$ for $-\infty \leq x \leq \infty$		Mean $= \mu$; Variance $= \sigma^2$ Most common distribution. Used unless another distribution is more applicable.
Lognormal	$f(x) = \dfrac{\exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu_y}{\sigma_y}\right)\right]}{x\,\sigma_y\,\sqrt{2\pi}}$ where $y = \ln x$ for $0 \leq x \leq \infty$		The random variable $y = \ln x$ is normally distributed.
Exponential	$f(x) = \lambda \exp[-\lambda\, x]$ for $0 \leq x \leq \infty$		Mean $= 1/\lambda$; Variance $= 1/\lambda^2$ Used for particular physical situations when positive values required, e.g., lengths of joints in a rock mass. Also used to describe the time between incidents of events which can be described by a *Poisson* distribution (such as earthquakes and floods). See Benjamin and Cornell (1970).
Beta	$f(x) = \dfrac{(x - a)^{\sigma-1}(b - x)^{\beta-\alpha-1}}{B(b - a)^{\beta-1}}$ where B (beta function) $= \dfrac{\Gamma(\alpha)\Gamma(\beta - \alpha)}{\Gamma(\beta)}$ $\Gamma(\ ) =$ gamma function for $a \leq x \leq b$	 	Mean $= a + \dfrac{\alpha}{\beta}(b - a)$ Variance $= (b - a)^2\,\dfrac{\alpha(\beta - \alpha)}{\beta^2(\beta + 1)}$ Extremely versatile distribution for matching data over the range [a,b]. Variation of parameters α and β gives wide variety of shapes. Contains as special cases the uniform and normal distributions. Can be symmetrical or skewed right or left. See Benjamin and Cornell (1970), and Harr (1977).

Table 4 -- Cumulative frequencies of the Normal distribution
(from Benjamin and Cornell, 1970).

Cumulative Probabilities of the Normal Probability Distribution* (areas under the normal curve from $-\infty$ to z)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
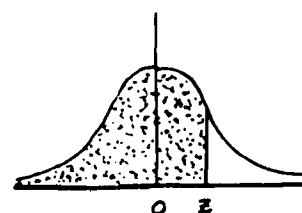
z	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.891	4.417
$F(z)$	.90	.95	.975	.99	.995	.999	.9995	.99995	.999995
$2[1 - F(z)]$	.20	.10	.05	.02	.01	.002	.001	.0001	.00001

PLATE 1

SUBJECT: Example calculations of Summary statistics for (R-test) data.

I. PROBLEM:

Calculate summary statistics from experimental R-test data on soil strength.

II. SOLUTION:

1. Measured data:

Measured values of R-test data as shown in Figure 3.

2. Measures of central tendency:

mean --	m_x	$= (1/n) \; \Sigma x_i = 32.6\%$
median --	$x_{0.5}$	$= 32.1\%$
mode --	x_0	$= 28.5\%$

3. Measures of dispersion:

standard deviation --
$$s_x = \frac{1}{n-1} \; \Sigma (x_i - m_x)^2 = 6.5\%$$

variance --
$$V_x = s_x^2 = 42.8\%$$

range --
$$r = (x_{max} - x_{min}) = 29\%$$

fractiles --
$$x_{0.25} = 27.4\%$$
$$x_{0.50} = 32.1\%$$
$$x_{0.75} = 36\%$$

interquartile range --
$$r_{0.5} = (x_{0.75} - x_{0.25}) = 8.6\%$$

PLATE 2

SUBJECT: Shortcut estimates of summary statistics.

I. PROBLEM:

Estimate summary statistics using shortcut methods and compare to accurate calculations.

II. DATA:

Measured Strength (kPa): 38, 51, 43, 39, 48, 45, 42, 45, 49.

III. ESTIMATE MEAN:

Shortcut Method Using Median

$$m_x \approx \text{median of } x_i$$
$$= \underline{45 \text{ kPa}}$$

By Equation 2

$$m_x = \frac{1}{n} \Sigma \, x_i = \frac{1}{9} (400 \text{ kPa}) = 44.4 \text{ kPa}$$

IV. ESTIMATE STANDARD DEVIATION:

Shortcut Method Using Range

$$w = (x_{max} - x_{min})$$
$$= 51 - 38 \text{ kPa}$$
$$= 13 \text{ kPa}$$

N from Table 1 (for n=9): 0.337

$$s_x \approx (0.337) (13)$$
$$= 4.4 \text{ kPa}$$

By Equation 3

$$s_x = \frac{1}{n-1} \Sigma \, (x_i - m_x)^2 = 4.2 \text{ kPa}$$
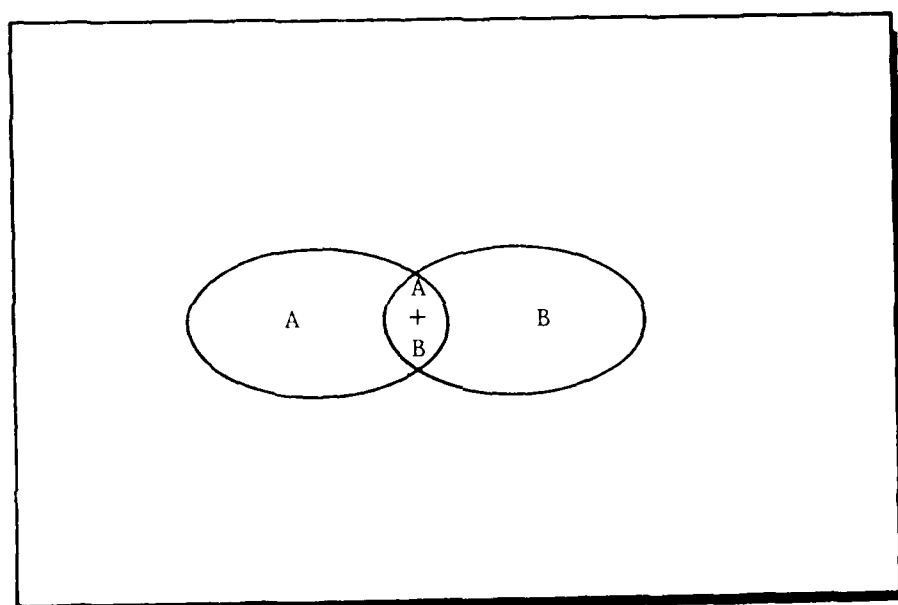
Figure 1 -- Venn diagram showing relations among probabilities of simple events.

TOTAL NUMBER OF DATA ·   235
MEAN · 8.9021   STD DEV · 4.4283
NUMBER OF OUTLIERS BEYOND LOWER LIMIT OF PLOT ·   0
NUMBER OF OUTLIERS BEYOND UPPER LIMIT OF PLOT ·   0
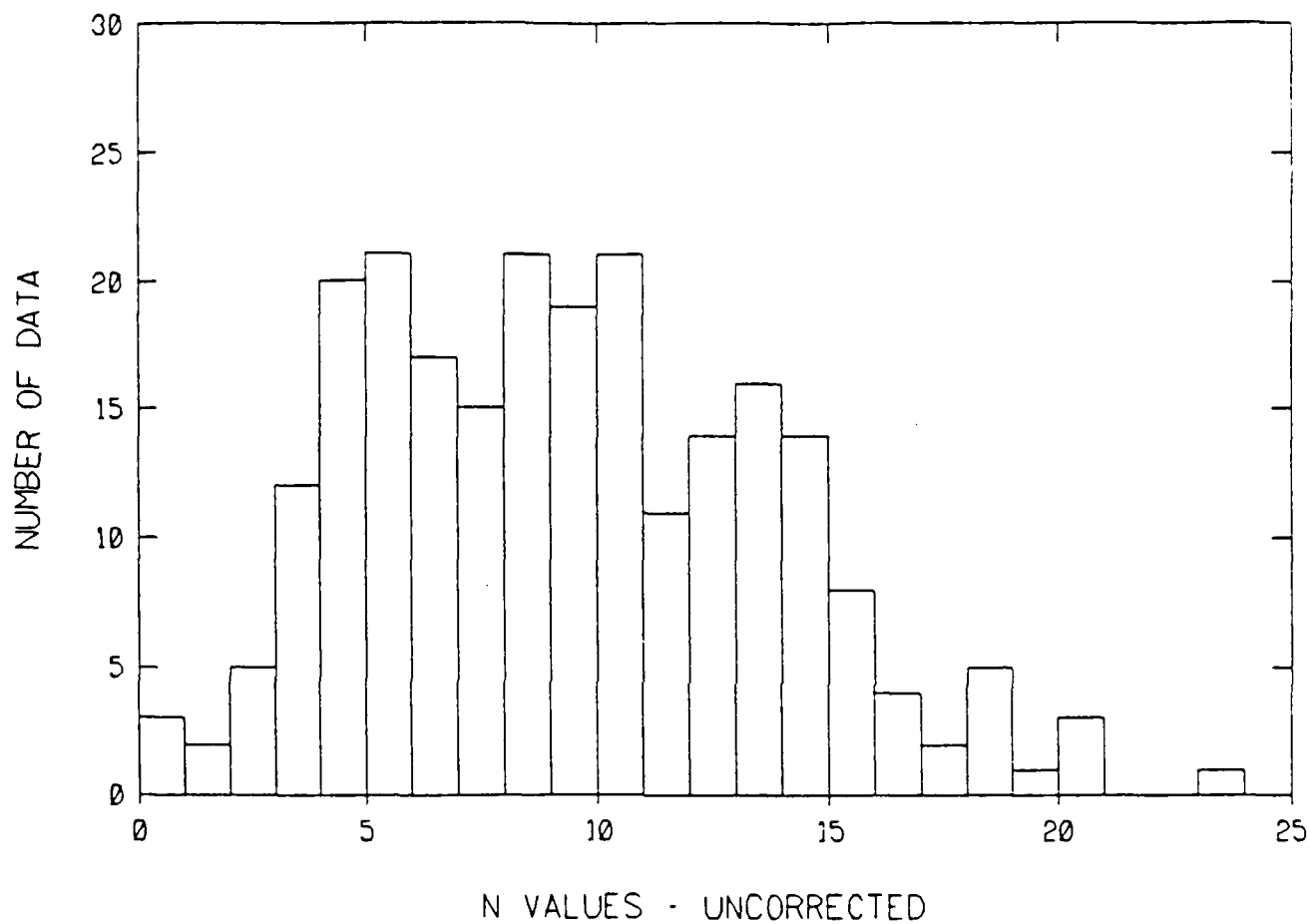
Figure 2 -- Histogram of SPT data.

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Figure 3 -- Histogram of R-test data.

TOTAL NUMBER OF DATA · 235
MEAN · 8.9021   STD DEV · 4.4283
NUMBER OF OUTLIERS BEYOND LOWER LIMIT OF PLOT ·   0
NUMBER OF OUTLIERS BEYOND UPPER LIMIT OF PLOT ·   0
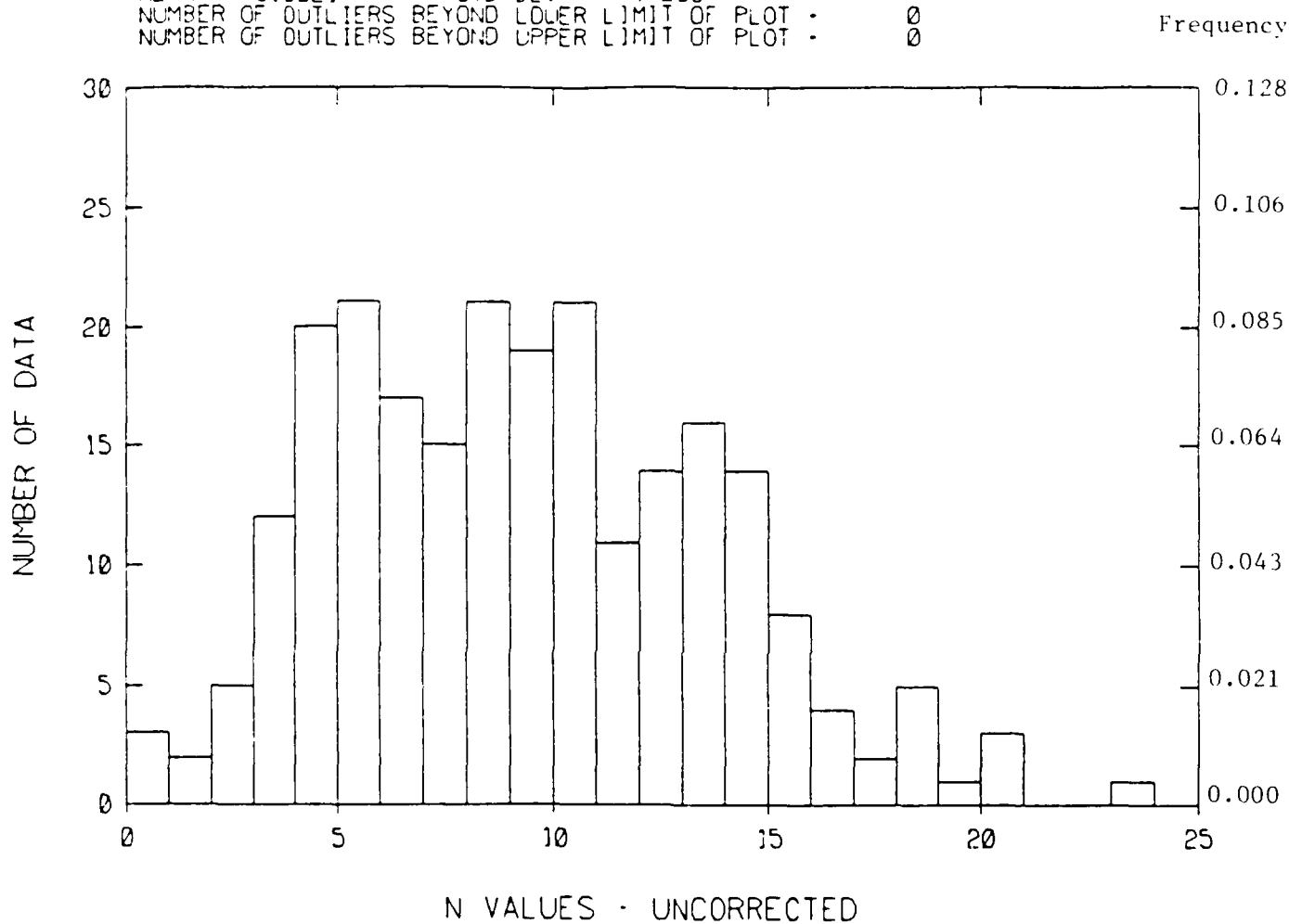
Figure 4 -- Frequency distributions for the data of Fig. 2.

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TOTAL NUMBER OF DATA · 73
MEAN · 32.591   STD DEV · 6.5427
NUMBER OF OUTLIERS BEYOND LOWER LIMIT OF PLOT · 0
NUMBER OF OUTLIERS BEYOND UPPER LIMIT OF PLOT · 0

WATER CONTENT @ FAILURE

RE-REG DAM FOUND. SOILS "R" TESTS
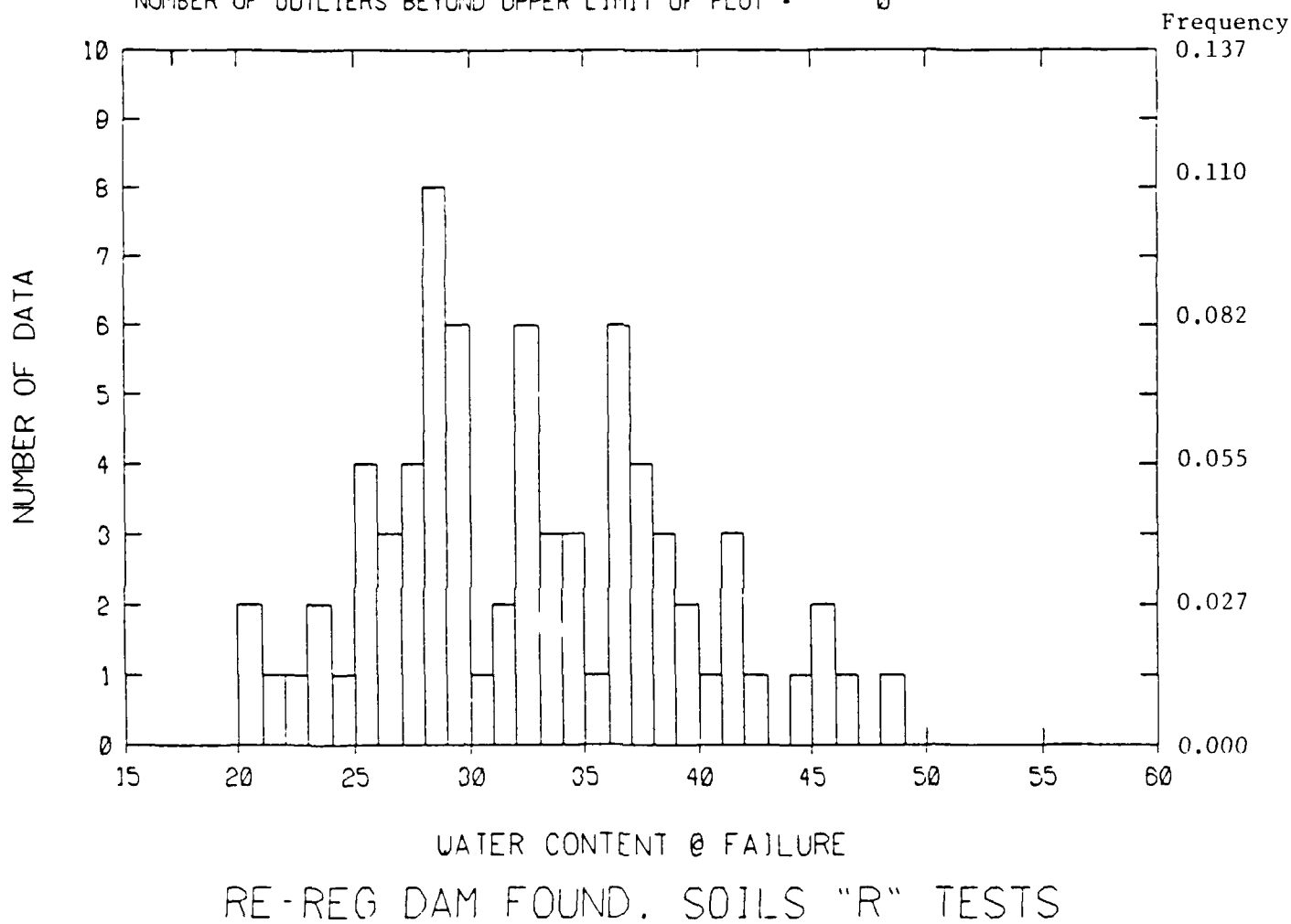
Figure 5 -- Frequency distributions for the data of Fig. 3.

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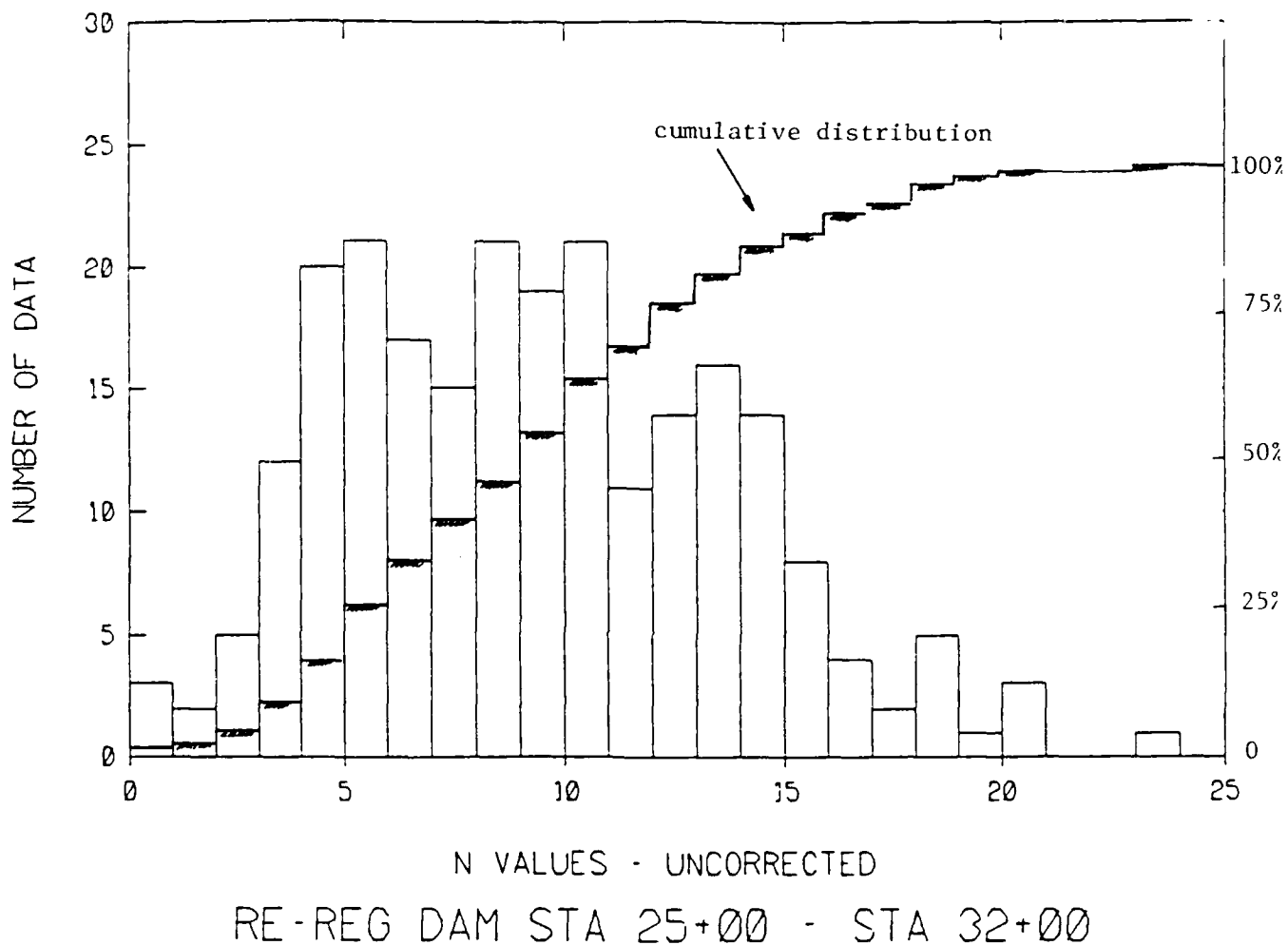
Figure 6 -- Cumulative distribution of the SPT data.
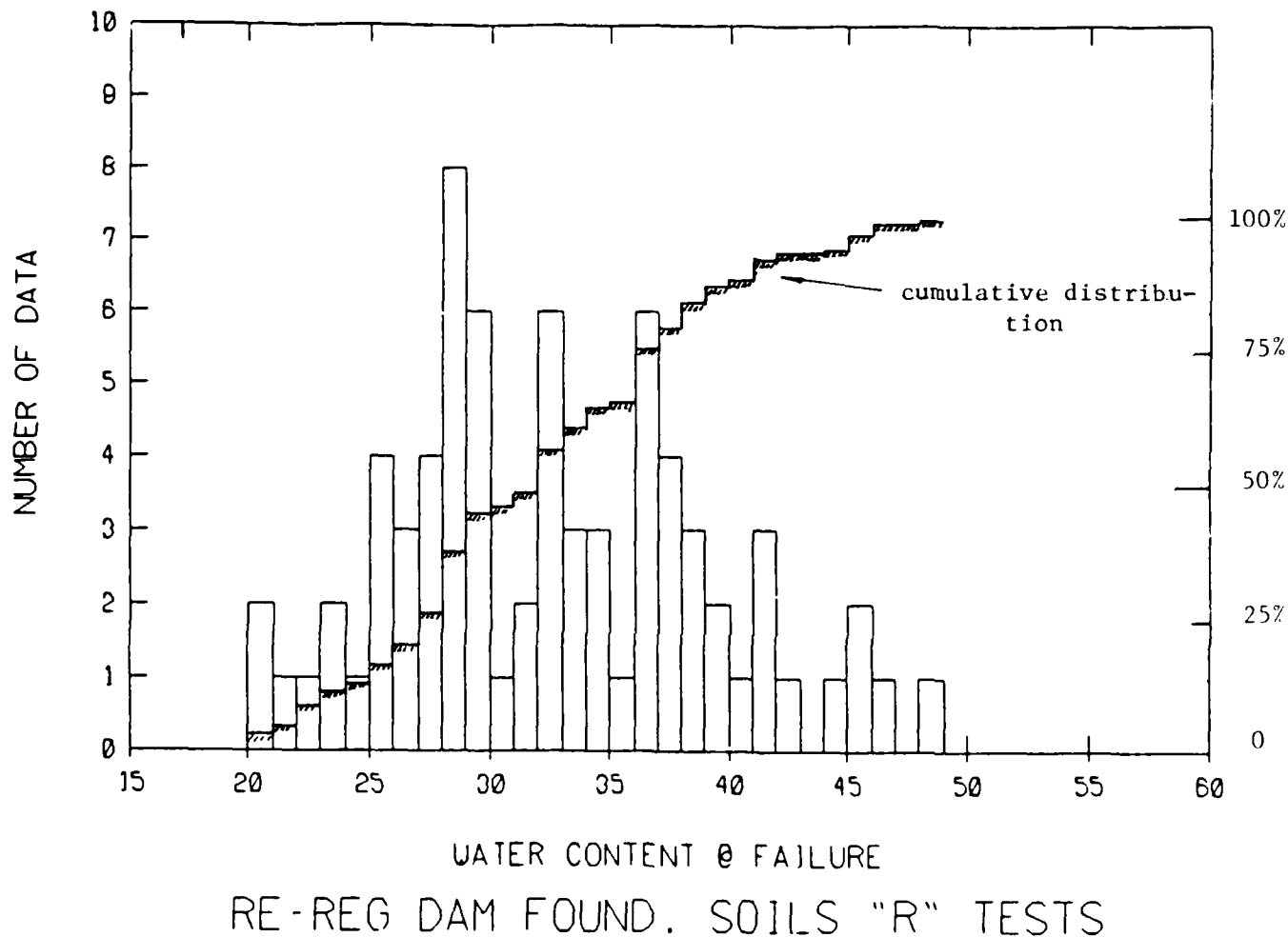
Figure 7 -- Cumulative distribution of the R-test data.

(a) symmetric
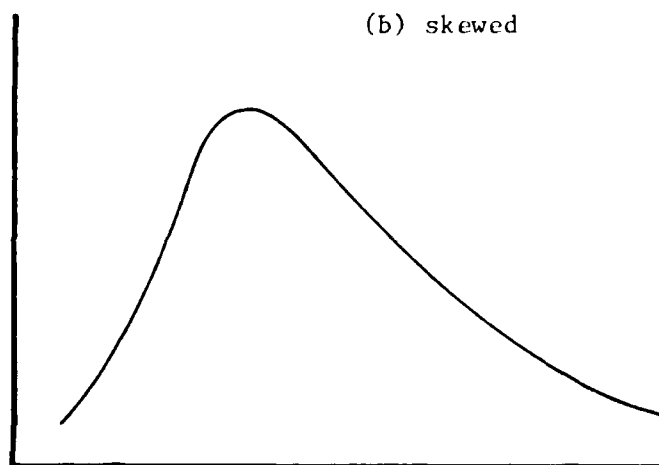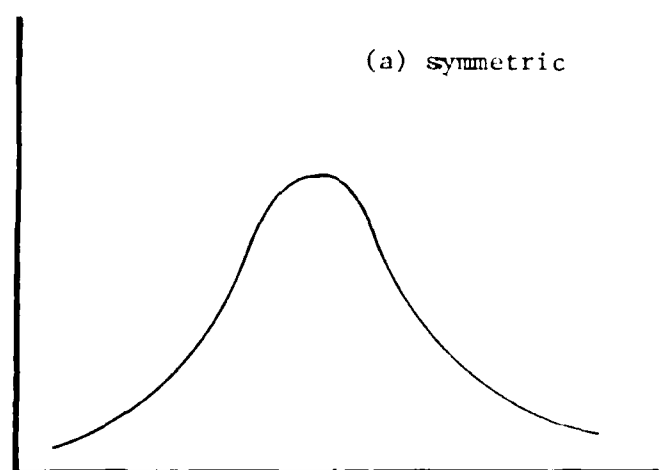
(b) skewed

Figure 8 -- Symmetric and skewed frequency distributions.
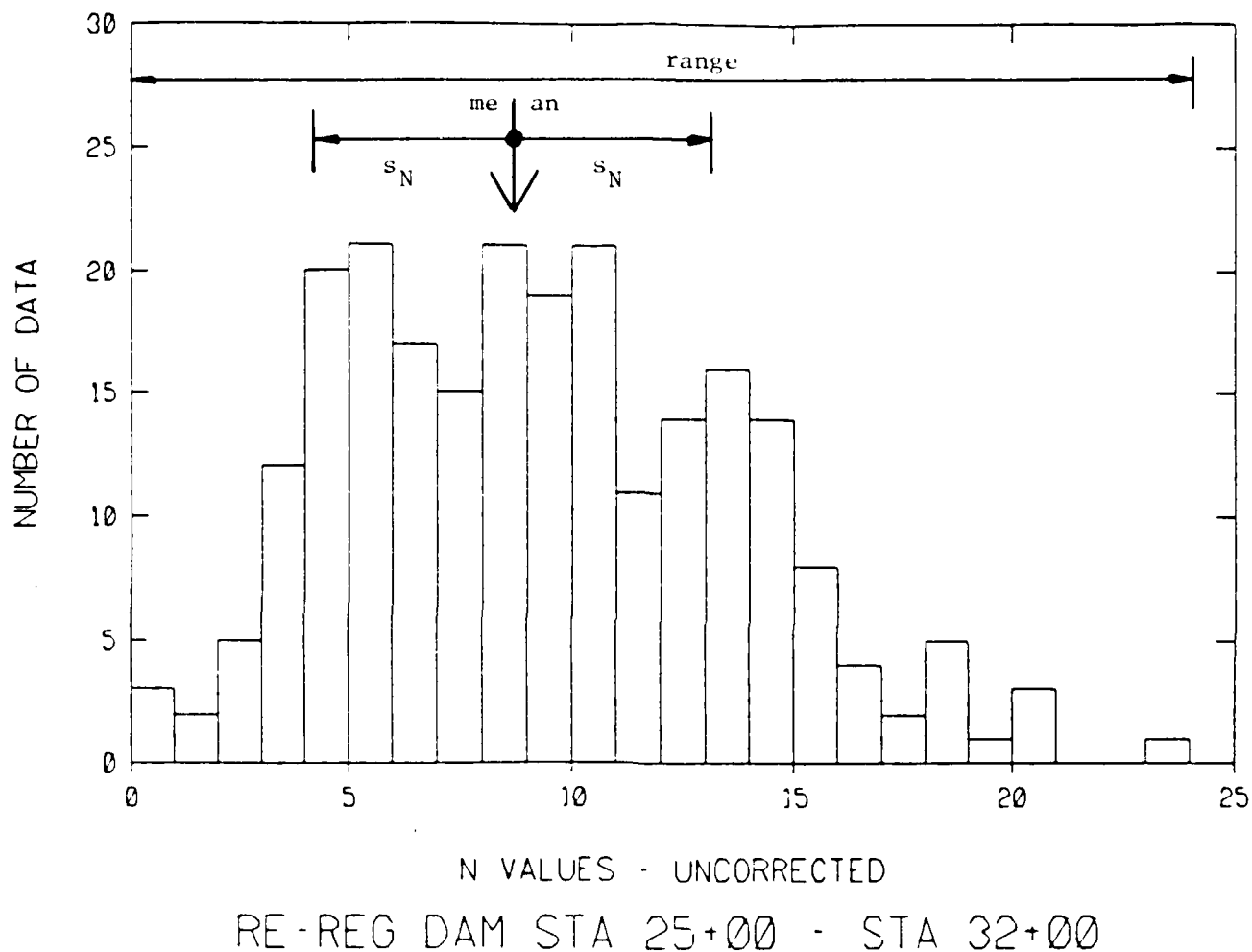
Figure 4 -- Summary statistics of a frequency distribution.

SHILLING'S METHOD:

$$r_{x,y} = (12.0^2 - 4.0^2)/(12.0^2 + 4.0^2)$$

$$= 0.80$$
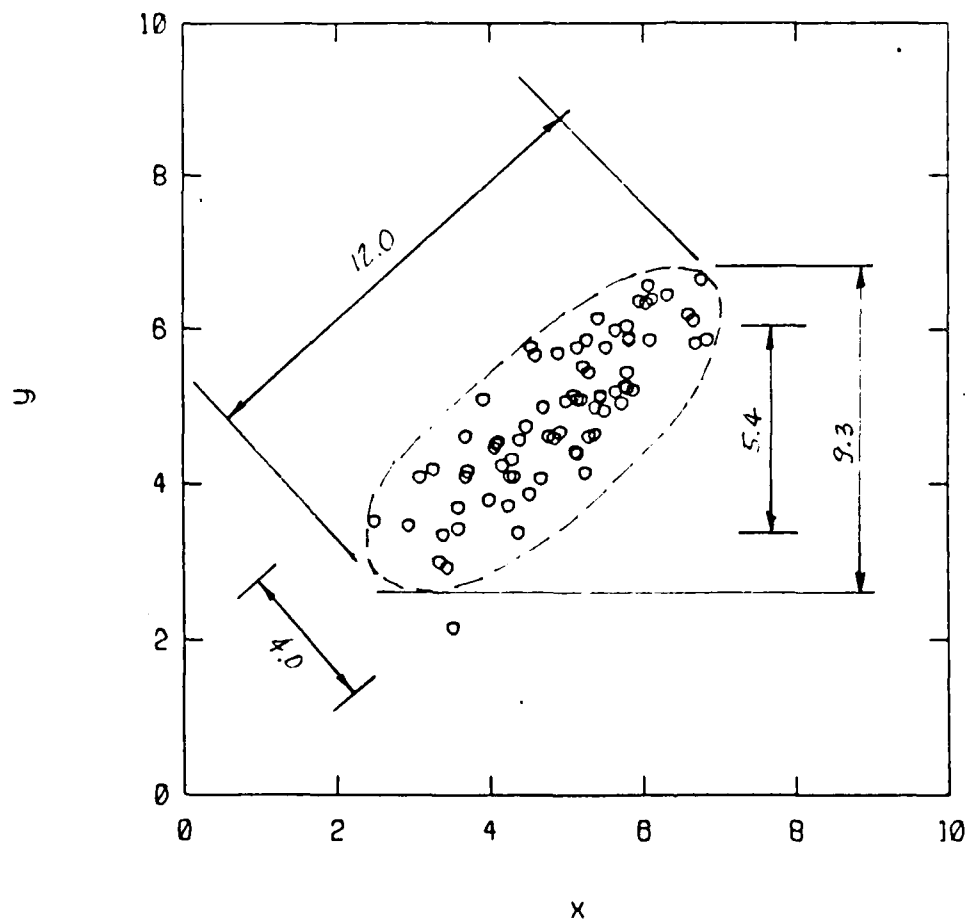
CHATILLON'S METHOD:

$$r_{x,y} = \sqrt{1 - (5.4/9.3)^2}$$

$$= 0.81$$

Figure 10 -- Example application of balloon method for estimating the correlation coefficient of experimental data.
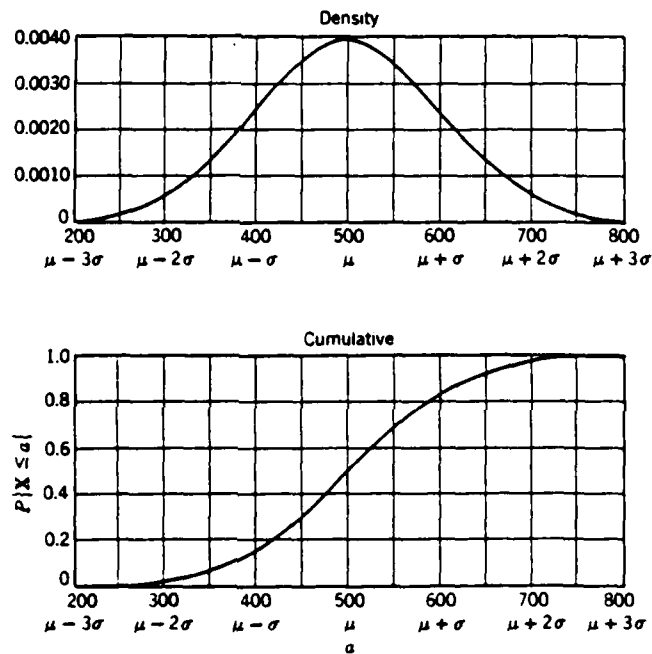
Figure 11 -- Normal or bell-shaped frequency distribution.
(from Chernoff and Moses, 1950, Elementary
Decision Theory, John Wiley and Sons, NY)

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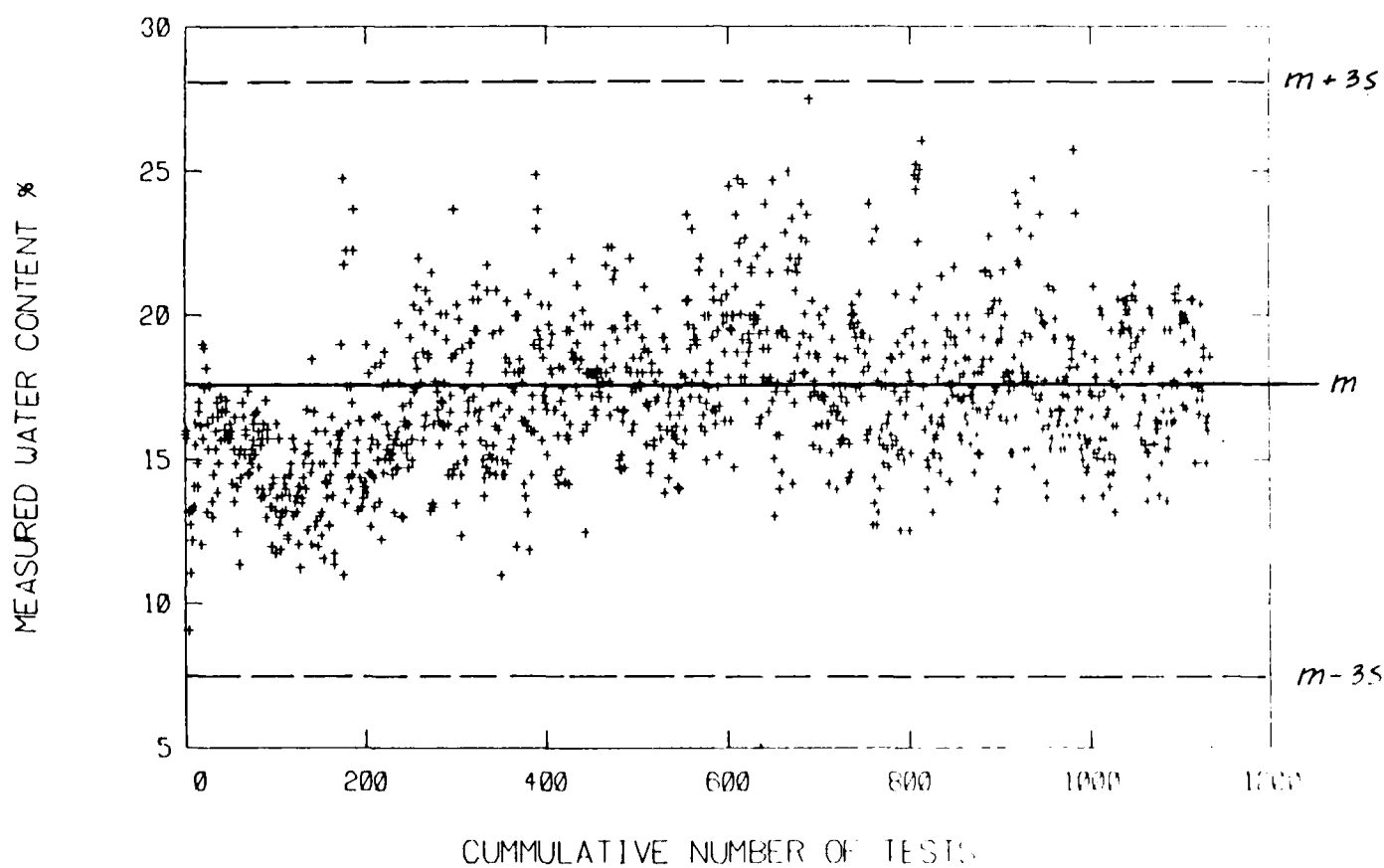
Figure 12 -- Typical control chart for the output of a process operating in a random manner.

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Part III: CONCEPTS OF STATISTICAL QUALITY CONTROL

Quality Assurance and Quality Control

The terms quality assurance (QA) and quality control (QC) are used in special and differing ways by different organizations. In this report,

Quality assurance means an inspection program aimed at assuring that soils placed in a fill meet specifications.

Quality control means an inspection program aimed at monitoring construction performance to give early warning of changes that affect quality and thus to provide a basis for controlling the process.

Quality assurance programs prescribe a procedure which when consistently applied to inspection data yield a specified risk of accepting lifts of given quality. A QA program provides a decision procedure. Quality control, on the other hand, provides a way of estimating lift properties and the changes in those properties with time. A QC program provides a monitoring scheme. QA provides a rule by which the owner's risk of accepting poor quality construction is guaranteed and balanced against the contractor's risk of having good quality construction rejected. QC provides a tool by which owner and contractor alike can make efforts to maintain a uniformly high quality product.

Sampling

Measurements are made on a set of soil specimens or at a set of locations in order to estimate the properties of a soil deposit or an engineered structure as a whole. Statisticians call this set of measurements a sample.

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An individual piece of soil is called a specimen to distinguish it from the concept of a statistical sample.

The soil deposit or structure whose properties are of interest is called the target population (Fig. 13). A population in statistics is simply a large (or infinite) collection of elements. Not all of the elements in the target population may be accessible for sampling. Those that are accessible are said to compose the sampled population. From this sampled population a finite number of elements are selected for testing and this set is called a sample. If the way this sample is chosen satisfies certain rules, the sample is said to be a probability sample. Statistical methods can the be used to quantify the uncertainty in estimates from the sample about properties of the sampled population. Statistics is powerless to say anything about the correspondence between sampled and target populations, however, as this is a geological or engineering question.

Scientific Sampling

The concept of scientific sampling, or probability sampling, is central to quality assurance and control. A scientific sample is planned according to statistical principles. The importance of scientific sampling is that it allows quantitative statements about the uncertainty in parameter estimates which result from sampling. Other sampling schemes—as for example, instructing an inspector to purposely seek out areas in a fill that appear poorly compacted—certainly have merit in special circumstances, but they do not allow the quantitative analysis which has come to underlie modern engineering practice.

To be a probability sample, three criteria must be satisfied: (1) sample points must be chosen randomly, (2) all elements in the sampled population must have a non-zero chance of being sampled, and (3) different probabilities of each element being sampled must be compensated by weighting.
If these two criteria are satisfied--and only if they are satisfied-- statistical methods can be used to determine uncertainties properly associated with parameter estimates. This means that for statistical methods to be used, some form of random sampling is necessary. Purposive sampling, by which an inspector consciously selects for testing those elements that appear of poor quality, is intuitively appealing and can provide important information, but it cannot form the basis for statistical quality control. From a purposive sample there is, (a) no way to assign quantitative confidences to estimates of soil parameters, (b) no way to explicitly review an inspection program after the fact, and (c) no way to establish a defendable level of quality assurance.

People also talk about having an inspector seek out a 'representative' sample. This, too, may have merit in special circumstances, but it does not produce a sample from which quantitative conclusions can be drawn. No individual sample is representative of a sampled population. A sample contains specific measurements which can never precisely mirror the subtlety of variations in the sampled population. On the other hand, a sampling plan can be made representative, if designed by scientific principles, in that it affords every element within the sampled population an equal chance to influence estimates that are made.

Random Sampling

Scientific sampling requires that every element in the sampled population have a non-zero chance of appearing in the sample. It does not require these chances to all be the same, only that the relative probabilities are known. This condition requires that elements be selected from the sampled population in a random way. Lacking a random procedure, the assumption that each element has a non-zero chance of being sampled cannot be made, and the relative probabilities of different elements being sampled cannot be assessed. The use of a random procedure attempts to avoid any form of association between the selection of elements for the sample and the properties of the elements that are being sampled. Such association is called a bias.

Randomization means selecting elements of a sample in such a way that the two conditions of probability sampling are satisfied. Randomization can be accomplished many ways. A conceptually simple but operationally clumsy way is to pick sampling locations by a random number generator or table of random numbers (Table 5). If performed faithfully this scheme gives each element in the sampled population an equal chance of being sampled. Another way to provide randomization is to layout measurements on a fixed grid and then randomly locate the first point.

Sampling Plans

An essentially infinite number of sampling plans for quality assurance or quality control satisfy the properties of probability sampling. These are all randomized sampling plans in the sense that the exact elements which are sampled depend on the outcome of some chance event.

A more convenient sampling plan is to layout sample points on a grid, and then locate the grid in the field by randomly selecting its first point (Fig. 14b). Only one pair of random numbers needs to be chosen from which all of the sample points are determined. The disadvantage of a grid plan compared to purely random plans is that any spatial periodicity in the compaction process may bias the outcome. An advantage compared to the purely random plan, especially with small sample sizes, is that uniform coverage of the site is assured.

To provide coverage while at the same time limiting the possible effects of periodicities, stratified random sampling plans are sometimes used. Using a stratified plan the sampled area is first divided into a regular array of squares or rectangles (Fig. 14c) and then a sample point is randomly located in each.

Another common plan is nested sampling. Nested sampling uses a pre-fixed grid of sample points with varying spacings (Fig. 14d). The first point is located randomly as in grid sampling and from that point all the rest are specified. The principal use of nested sampling is for estimating special aspects of the spatial structure of soils data, namely the autocorrelation function or variogram, (see the report, "Statistical analysis of geotechnical data", Instructional Report GL-87). The use of nested sampling in quality control or quality assurance of compacted fills is mostly for the purpose of assessing measurement errors or noise in the resulting data.

Random clumped sampling plans are often used whenever a large spatial extent must be sampled, or when the cost of mobilization and demobilization

operation is large compared to the incremental cost of testing. Clumped sampling involves two stages. In the first stage a number of seed points are randomly chosen. In the second stage a number of sampling points are chosen in the vicinity of each seed point. At both the first and second stage the sampling plan can be purely random, gridded, stratified, nested, or so forth.

The sampling plans reviewed here are typical of the very large number of possible sampling places. In practical situations the constraints of a particular project may dictate that a specialized plan be developed. This is accepted practice as long as the principles of probability sampling are adhered to. These principles dictate three things, (1) that sample points be chosen according to some random process and not be affected by the intuition of an inspector, (2) that all elements within the population to be sampled have a non-zero chance of being sampled, and (3) that if the probabilities of each element being sampled are not all the same, these differences in probability be appropriately compensated for by weighting when the data are analyzed.

For most quality control and quality assurance sampling in geotechnical engineering the probabilities of elements within the sampled population being sampled are all the same. Therefore, for these sampling plans the problem of weighting sample outcomes is seldom of concern. For those cases where weighting is necessary, Cochran (1964) provides techniques and practical suggestions.

Sampling Variation

When a sample of n specimens is collected and tested, specific numerical data are obtained. Let these n test results be denoted $x_1, \ldots, x_n$. Obviously,

a single test may yield more than one numerical result--for example, water

content, dry density, plasticity index, and so forth--but for now nothing is

lost by considering only a single scalar outcome.

If another sample of n specifications is now taken from the same lift of

soil, however with the specimens taken at slightly different places, another

set of n numerical data will result. Each of these will differ somewhat from

their counterparts in the first sample, because the soil itself varies from one

spot to another and because there are a number of instrument or operator

effects which influence test results. This variation in numerical results from

one sample to another is called sample variation. Statistical techniques allow

such sample variation to be predicted and dealt with in a quantitative way.

The sample mean (Eqn. 11), sample standard deviation (Eqn. 13), and other

summary measures calculated from the test results $x_1, \ldots, x_n$ are simply

mathematical function of the data. If the data vary from one sample to

another, so will the summary measures.

Sampling Variability of the Mean

The sample mean m_x is calculated by Eqn. 11. If many tests are made

(i.e., if n is large), variations in one test result within an average will be

offset by variations in others, and as a result m_x should be fairly close to

the actual mean of the sampled population m_x. In this report the actual

sampled populations mean is denoted by a prime, m_x', as compared to the sample

mean which is denoted without a prime, m_x. On the other hand, if few tests are

made (i.e., n is small), variation in test results will not have as much

opportunity to average out, and as a result the sample mean may deviate

considerably from m_x'. This sampling variability is the critical factor in

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deciding how many tests must be made in a quality control or quality assurance program.

If the standard deviation of the sampled population is known and if the individual measurements are independent of one another, then the means of individual samples each of size n will vary with a standard deviation of

$$s_{m_X} = s_X / \sqrt{n}. \hspace{3cm} -25-$$

For example, Fig. 15 shows a histogram of sample means, each corresponding to a different sample of size n = 5 taken randomly from the SPT blow count data in Fig. 2. The standard deviation of the sampled set of data is 4.4 bpf, while the standard deviation of the variability of the sample means is 2 bpf $\approx$ 4.4 bpf/ n. If plotted as a frequency distribution, the variability of the sample means will be approximately Normally distributed, almost without regard to the shape of the frequency distribution of the sampled population. The approximation to the Normal distribution becomes better as n becomes larger.

In the more common case the true standard deviation of the sampled population, s_X', is not known, and thus the sample standard deviation, s_X, is used in equation 25 to approximate the variability of m_X about m_X'. Using s_X rather than s_X' underestimates the variability in m_X, however, because the estimate of s_X differs somewhat from the true standard deviation s_X'. To overcome this limitation a standardized mean is used,

$$t = \frac{m_X - m_X'}{s_X} \hspace{3cm} -26-$$

56

in which m_x and s_x are the sample mean and sample standard deviation, and m_x'
is the true mean of the sampled population. If the value t is estimated
separately from a large number of samples, each sample containing the same
number of observations, the frequency distribution of t over these many
separate samples will have a standard deviation of 1.0 and a shape known as
Student's-t distribution. The Student's-t distribution looks much like a
Normal distribution, but with thicker tails and a higher mode. That is, the
Student t has somewhat more of what statisticians call kurtosis than a Normal
distribution does. Areas under the Student curve are given in Table 6, and may
be approximated by series expansions, as given by (Abramowitz and Segun (1964).
The shape of the Student's-t distribution and thus the areas beneath it depend
on the number of measurements within a sample, n. This enters Table 6 as the
degrees-of-freedom parameter $\upsilon = n-1$.

Sampling Variability of the Standard Deviation

Just as the sample mean varies from one sample to another, so do other
summary measures such as the sample standard deviation or sample variance.
Unfortunately, the statistical results for the variability if the sample
standard deviation and variance are not as simple as those for the sample mean.
For samples taken from Normally distributed data, the sample standard deviation
varies approximately with a standard deviation of,

$$s_{s_X} \approx s_X / \sqrt{2n} \quad , \qquad\qquad -27-$$

57

in which n = the sample size (Snedecor and Cochran, 1980). The sample variance, s_x^2, varies approximately with a standard deviation of,

$$s_{s_x^2} \approx s_x^2 \sqrt{\frac{2}{n-1}} \quad . \qquad\qquad -28-$$

Similar results are available for non-Normally distributed data, but are more complicated. Most basic statistics textbooks discuss these results (e.g., Snedecor and Cochran, 1980).

Sampling Variability of the Range

Because the sample range $r_x = |x_{max} - x_{min}|$ is easier to calculate than the standard deviation, it is often preferred as a measure of variablity in programs of quality control. For a sample taken from Normally distributed data the frequency distribution of the relative range

$$w_x = r_x/s_x \qquad\qquad -29-$$

across samples of size n is tabulated in Table 7.

In-Control vs. Out-of-Control Processes

The above discussions are based on the concept of a construction process operating in a random manner. When a process is operating in a random manner any part of its output may be viewed as typical of the output as a whole and perturbations in the process show no discernable pattern. A construction process operating in a random manner and producing few sample outcomes which deviate substantially from its average output is said to be "in-control." Fig.

16 shows a chart of compaction data for a process operating in-control. The variations in these data appear to behave randomly without trend or pattern.

Few construction processes operate in-control for significant lengths of time, and even when a process is in-control minor deviations from randomness have to be overlooked. The statistical theory of quality control and quality assurance is based on the idea of randomness in process output, and thus every effort should be made to assure that non-random factors are not present.

Whether a process is in-control also depends on the level of detail with which the output is scrutinized. Obviously, variations in the output of a construction process are not truly random. At some level of detail are all explainable by physical arguments. The notions of randomness and a process being in-control have to do with engineering decisions and the cost effective-ness of further reduction in output variability. Attempts are made to identify and eliminate all major sources of variability, and what remains and is not cost effective to further reduce is operationally handled as if it were random variation. As long as our statistical models can be successfully used to portray this residual variability and to characterize uncertainties which arise from it, then the process is for engineering purposes "in-control."

When the variability in the output of a construction process deviates from randomness, that is, when significant trends or patterns begin to appear in the output, statistical models no longer adequately capture the important features of the variations and the construction process is said to be out-of-control. The principal use of the control charts of Section 5 is to obtain early warning that a process is going out-of-control, and to identify steps that might be

taken to bring the process back in-control. Examples of changes that could cause a fill and compaction process to go out-of-control include a change in borrow materials; change in rainfall; change in construction superintendent, operator, or equipment; and so on.

Table 5 -- Table of uniform [0,1] random numbers
(from Cochran, 1977).

```
10 27 53 96 23   71 50 54 36 23   54 31 04 82 98   04 14 12 15 09   26 78 25 47 47
28 41 50 61 88   64 85 27 20 18   83 36 36 05 56   39 71 65 09 62   94 76 62 11 89
34 21 42 57 02   59 19 18 97 48   80 30 03 30 98   05 24 67 70 07   84 97 50 87 46
61 81 77 23 23   82 82 11 54 08   53 28 70 58 96   44 07 39 55 43   42 34 43 39 28
61 15 18 13 54   16 86 20 26 88   90 74 80 55 09   14 53 90 51 17   52 01 63 01 59

91 76 21 64 64   44 91 13 32 97   75 31 62 66 54   84 80 32 75 77   56 08 25 70 29
00 97 79 08 06   37 30 28 59 85   53 56 68 53 40   01 74 39 59 73   30 19 99 85 48
36 46 18 34 94   75 20 80 27 77   78 91 69 16 00   08 43 18 73 68   67 69 61 34 25
88 98 99 60 50   65 95 79 42 94   93 62 40 89 96   43 56 47 71 66   46 76 29 67 02
04 37 59 87 21   05 02 03 24 17   47 97 81 56 51   92 34 86 01 82   55 51 33 12 91

63 62 06 34 41   94 21 78 55 09   72 76 45 16 94   29 95 81 83 83   79 88 01 97 30
78 47 23 53 90   34 41 92 45 71   09 23 70 70 07   12 38 92 79 43   14 85 11 47 23
87 68 62 15 43   53 14 36 59 25   54 47 33 70 15   59 24 48 40 35   50 03 42 99 36
47 60 92 10 77   88 59 53 11 52   66 25 69 07 04   48 68 64 71 06   61 65 70 22 12
56 88 87 59 41   65 28 04 67 53   95 79 88 37 31   50 41 06 94 76   81 83 17 16 33

02 57 45 86 67   73 43 07 34 48   44 26 87 93 29   77 09 61 67 84   06 69 44 77 75
31 54 14 13 17   48 62 11 90 60   68 12 93 64 28   46 24 79 16 76   14 60 25 51 01
28 50 16 43 36   28 97 85 58 99   67 22 52 76 23   24 70 36 54 54   59 28 61 71 96
63 29 62 66 50   02 63 45 52 38   67 63 47 54 75   83 24 78 43 20   92 63 13 47 48
45 65 58 26 51   76 96 59 38 72   86 57 45 71 46   44 67 76 14 55   44 88 01 62 12

39 65 36 63 70   77 45 85 50 51   74 13 39 35 22   30 53 36 02 95   49 34 88 73 61
73 71 98 16 04   29 18 94 51 23   76 51 94 84 86   79 93 96 38 63   08 58 25 58 94
72 20 56 20 11   72 65 71 08 86   79 57 95 13 91   97 48 72 66 48   09 71 17 24 89
75 17 26 99 76   89 37 20 70 01   77 31 61 95 46   26 97 05 73 51   53 33 18 72 87
37 48 60 82 29   81 30 15 39 14   48 38 75 93 29   06 87 37 78 48   45 56 00 84 47

68 08 02 80 72   83 71 46 30 49   89 17 95 88 29   02 39 56 03 46   97 74 06 56 17
14 23 98 61 67   70 52 85 01 50   01 84 02 78 43   10 62 98 19 41   18 83 99 47 99
49 08 96 21 44   25 27 99 41 28   07 41 08 34 66   19 42 74 39 91   41 96 53 78 72
78 37 06 08 43   63 61 62 42 29   39 68 95 10 96   09 24 23 00 62   56 12 80 73 16
37 21 34 17 68   68 96 83 23 56   32 84 60 15 31   44 73 67 34 77   91 15 79 74 58

14 29 09 34 04   87 83 07 55 07   76 58 30 83 64   87 29 25 58 84   86 50 60 00 25
58 43 28 06 36   49 52 83 51 14   47 56 91 29 34   05 87 31 06 95   12 45 57 09 09
10 43 67 29 70   80 62 80 03 42   10 80 21 38 84   90 56 35 03 09   43 12 74 49 14
44 38 88 39 54   86 97 37 44 22   00 95 01 31 76   17 16 29 56 63   38 78 94 49 81
90 69 59 19 51   85 39 52 85 13   07 28 37 07 61   11 16 36 27 03   78 86 72 04 95

41 47 10 25 62   97 05 31 03 61   20 26 36 31 62   68 69 86 95 44   84 95 48 46 45
91 94 14 63 19   75 89 11 47 11   31 56 34 19 09   79 57 92 36 59   14 93 87 81 40
80 06 54 18 66   09 18 94 06 19   98 40 07 17 81   22 45 44 84 11   24 62 20 42 31
67 72 77 63 48   84 08 31 55 58   24 33 45 77 58   80 45 67 93 82   75 70 16 08 24
59 40 24 13 27   79 26 88 86 30   01 31 60 10 39   53 58 47 70 93   85 81 56 39 38

05 90 35 89 95   01 61 16 96 94   50 78 13 69 36   37 68 53 37 31   71 26 35 03 71
44 43 80 69 98   46 68 05 14 82   90 78 50 05 62   77 79 13 57 44   59 60 10 39 66
61 81 31 96 82   00 57 25 60 59   46 72 60 18 77   55 66 12 62 11   08 99 55 64 57
42 88 07 10 05   24 98 65 63 21   47 21 61 88 32   27 80 30 21 60   10 92 35 36 12
77 94 30 05 39   28 10 99 00 27   12 73 73 99 12   49 99 57 94 82   96 88 57 17 91
```

Table 6 -- Percentage points (i.e., double tail areas) of the Student-t
         distribution. For areas under a single tail, divide by two.
         From Duncan, 1974.
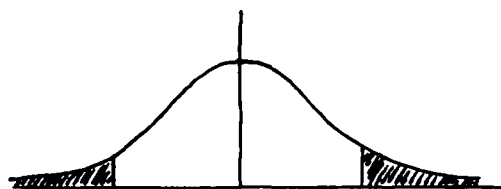
Probability (P).

ν	·9	·8	·7	·6	·5	·4	·3	·2	·1	·05	·02	·01	·001
1	·158	·325	·510	·727	1·000	1·376	1·963	3·078	6·314	12·706	31·821	63·657	636·619
2	·142	·289	·445	·617	·816	1·061	1·386	1·886	2·920	4·303	6·965	9·925	31·598
3	·137	·277	·424	·584	·765	·978	1·250	1·638	2·353	3·182	4·541	5·841	12·941
4	·134	·271	·414	·569	·741	·941	1·190	1·533	2·132	2·776	3·747	4·604	8·610
5	·132	·267	·408	·559	·727	·920	1·156	1·476	2·015	2·571	3·365	4·032	6·859
6	·131	·265	·404	·553	·718	·906	1·134	1·440	1·943	2·447	3·143	3·707	5·959
7	·130	·263	·402	·549	·711	·896	1·119	1·415	1·895	2·365	2·998	3·499	5·405
8	·130	·262	·399	·546	·706	·889	1·108	1·397	1·860	2·306	2·896	3·355	5·041
9	·129	·261	·398	·543	·703	·883	1·100	1·383	1·833	2·262	2·821	3·250	4·781
10	·129	·260	·397	·542	·700	·879	1·093	1·372	1·812	2·228	2·764	3·169	4·587
11	·129	·260	·396	·540	·697	·876	1·088	1·363	1·796	2·201	2·718	3·106	4·437
12	·128	·259	·395	·539	·695	·873	1·083	1·356	1·782	2·179	2·681	3·055	4·318
13	·128	·259	·394	·538	·694	·870	1·079	1·350	1·771	2·160	2·650	3·012	4·221
14	·128	·258	·393	·537	·692	·868	1·076	1·345	1·761	2·145	2·624	2·977	4·140
15	·128	·258	·393	·536	·691	·866	1·074	1·341	1·753	2·131	2·602	2·947	4·073
16	·128	·258	·392	·535	·690	·865	1·071	1·337	1·746	2·120	2·583	2·921	4·015
17	·128	·257	·392	·534	·689	·863	1·069	1·333	1·740	2·110	2·567	2·898	3·965
18	·127	·257	·392	·534	·688	·862	1·067	1·330	1·734	2·101	2·552	2·878	3·922
19	·127	·257	·391	·533	·688	·861	1·066	1·328	1·729	2·093	2·539	2·861	3·883
20	·127	·257	·391	·533	·687	·860	1·064	1·325	1·725	2·086	2·528	2·845	3·850
21	·127	·257	·391	·532	·686	·859	1·063	1·323	1·721	2·080	2·518	2·831	3·819
22	·127	·256	·390	·532	·686	·858	1·061	1·321	1·717	2·074	2·508	2·819	3·792
23	·127	·256	·390	·532	·685	·858	1·060	1·319	1·714	2·069	2·500	2·807	3·767
24	·127	·256	·390	·531	·685	·857	1·059	1·318	1·711	2·064	2·492	2·797	3·745
25	·127	·256	·390	·531	·684	·856	1·058	1·316	1·708	2·060	2·485	2·787	3·725
26	·127	·256	·390	·531	·684	·856	1·058	1·315	1·706	2·056	2·479	2·779	3·707
27	·127	·256	·389	·531	·684	·855	1·057	1·314	1·703	2·052	2·473	2·771	3·690
28	·127	·256	·389	·530	·683	·855	1·056	1·313	1·701	2·048	2·467	2·763	3·674
29	·127	·256	·389	·530	·683	·854	1·055	1·311	1·699	2·045	2·462	2·756	3·659
30	·127	·256	·389	·530	·683	·854	1·055	1·310	1·697	2·042	2·457	2·750	3·646
40	·126	·255	·388	·529	·681	·851	1·050	1·303	1·684	2·021	2·423	2·704	3·551
60	·126	·254	·387	·527	·679	·848	1·046	1·296	1·671	2·000	2·390	2·660	3·460
120	·126	·254	·386	·526	·677	·845	1·041	1·289	1·658	1·980	2·358	2·617	3·373
∞	·126	·253	·385	·524	·674	·842	1·036	1·282	1·645	1·960	2·326	2·576	3·291

Table 7 -- Percentage points of the distribution of relative range $w_x = r_x / s_x$ for small samples from Normal distributions (from Duncan, 1974).

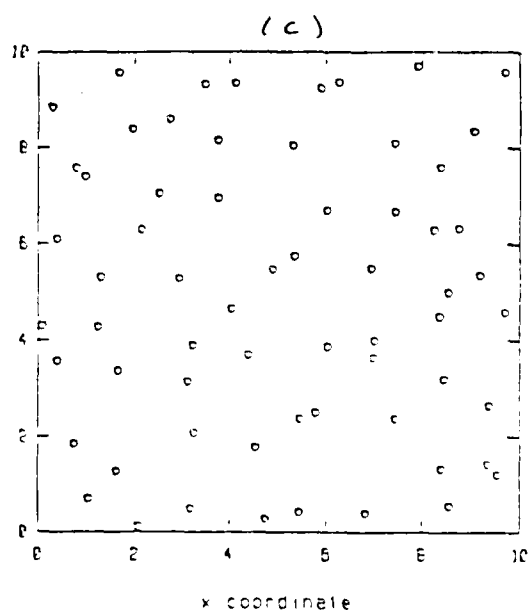| n	Mean w or d_{ℓ}	σ'_w or d_s	Probability That w Is Less than or Equal to Tabular Entry									
			0.001	0.005	0.010	0.025	0.050	0.950	0.975	0.990	0.995	0.999
2	1.128	0.8525	0.00	0.01	0.02	0.04	0.09	2.77	3.17	3.64	3.97	4.65
3	1.693	0.8884	0.06	0.13	0.19	0.30	0.43	3.31	3.68	4.12	4.42	5.06
4	2.059	0.8798	0.20	0.34	0.43	0.59	0.76	3.63	3.98	4.40	4.69	5.31
5	2.326	0.8641	0.37	0.55	0.66	0.85	1.03	3.86	4.20	4.60	4.89	5.48
6	2.534	0.8480	0.54	0.75	0.87	1.06	1.25	4.03	4.36	4.76	5.03	5.62
7	2.704	0.833	0.69	0.92	1.05	1.25	1.44	4.17	4.49	4.88	5.15	5.73
8	2.347	0.820	0.83	1.08	1.20	1.41	1.60	4.29	4.61	4.99	5.26	5.82
9	2.970	0.808	0.96	1.21	1.34	1.55	1.74	4.39	4.70	5.08	5.34	5.90
10	3.078	0.797	1.08	1.33	1.47	1.67	1.86	4.47	4.79	5.16	5.42	5.97
11	3.173	0.787	1.20	1.45	1.58	1.78	1.97	4.55	4.86	5.23	5.49	6.04
12	3.258	0.778	1.30	1.55	1.68	1.88	2.07	4.62	4.92	5.29	5.54	6.09
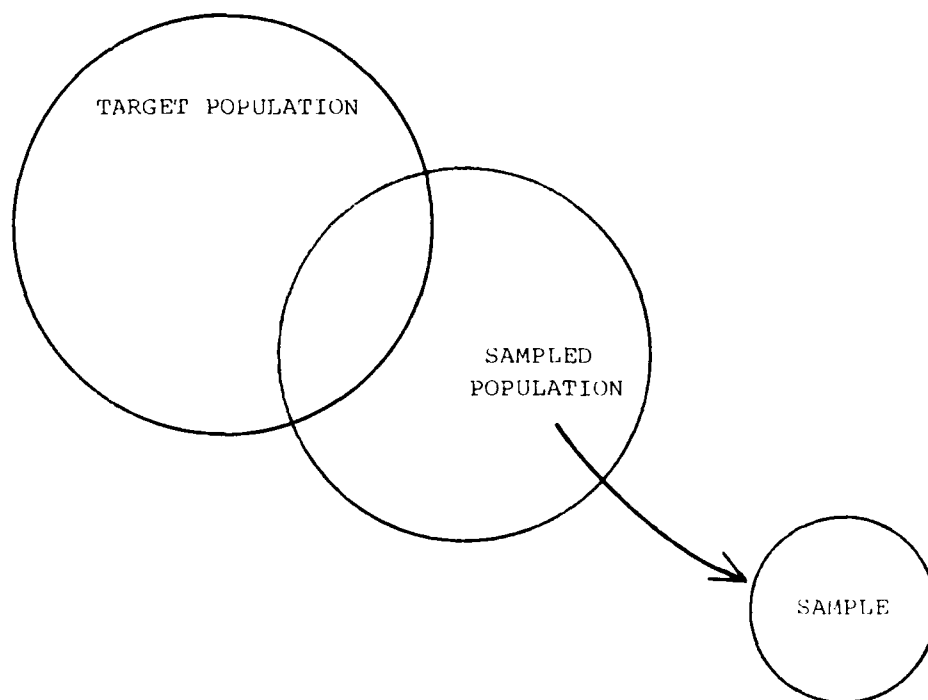
Figure 13 -- Populations of interest in sampling.

(a)
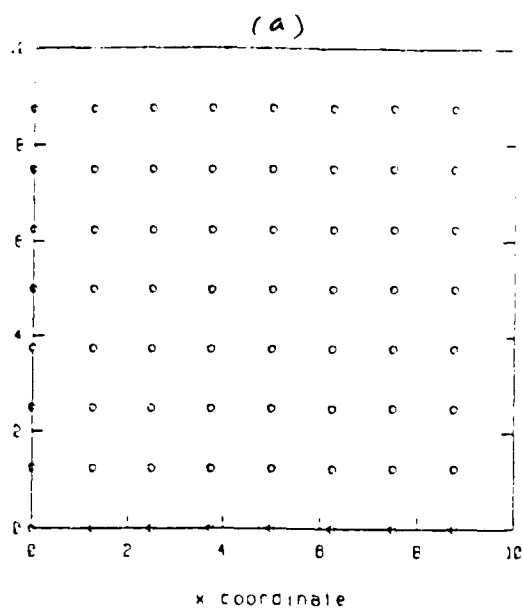
regular sampling plan n·64

(b)
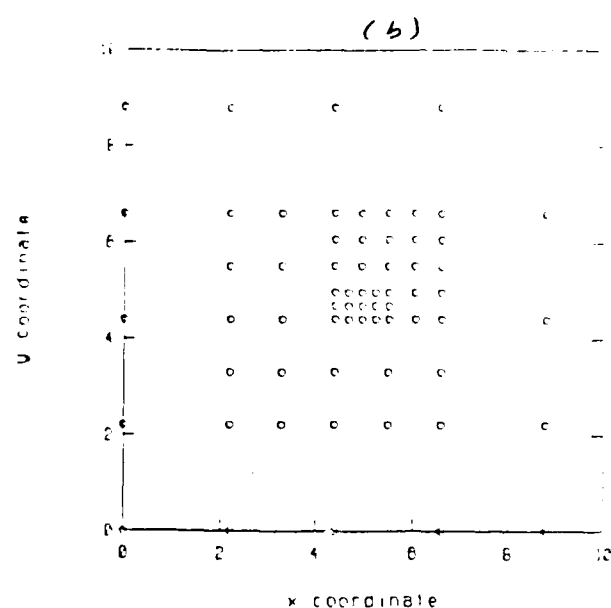
nested sampling plan n·66

(c)

sin. random sampling plan n·64

(d)
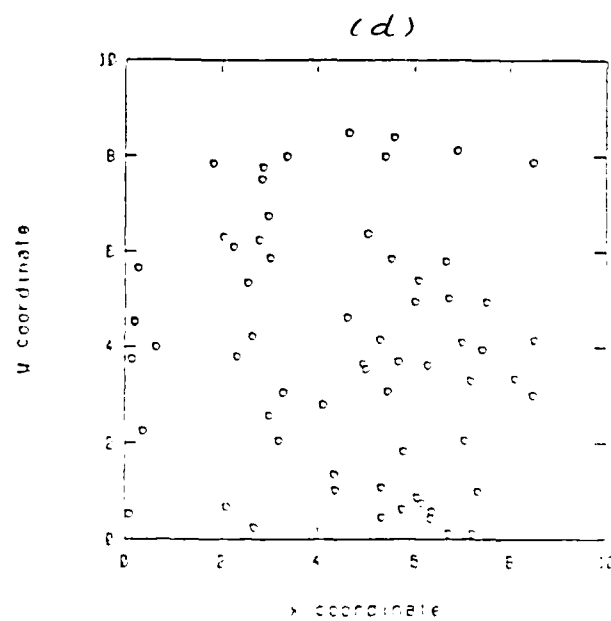
random sampling plan n·64
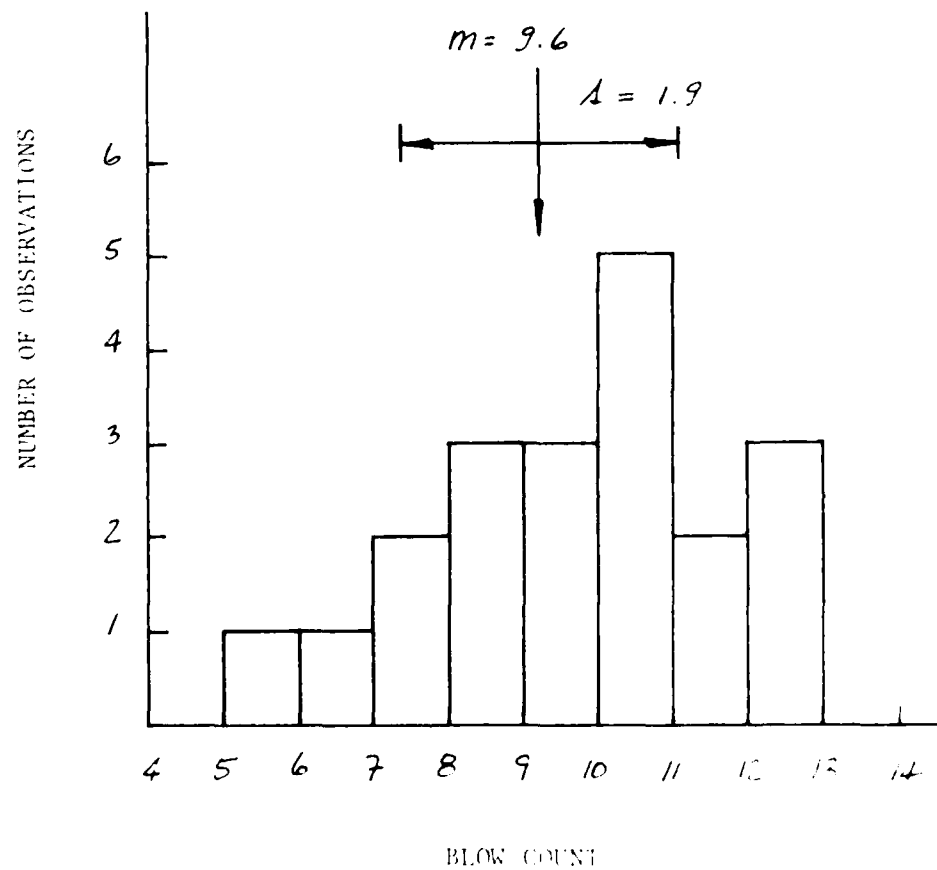
Figure 15 -- Histogram of the means of [illegible] of blow counts [illegible] sampled from a large data set. In all, [illegible] sets [illegible] data [illegible] were sampled.

PART IV -- INSPECTION OF ENGINEERED FILL

The construction of engineered embankments or fills must be controlled in order to ensure that zones within the fill are satisfactorily homogeneous and that average properties of the fill conform to specified requirements. Basic construction control is accomplished by visual inspection, complemented by a systematic program of sampling and testing aimed at verifying that the quality of materials placed in a fill is at least as good as that assumed in the design.

Visual examination during the process of placing and compacting provides an evaluation of the uniformity of soils placed in the fill, the thickness of lifts as placed, and the action of compaction equipment. Each of these provides justification of the adequacy of the construction program. When testing of the fill indicates a deficiency, the program must be presented accordingly.

Objectives of Sampling and Testing

The quantitative program of compaction inspection involving sampling and testing intends to establish whether compacted materials in a fill as placed satisfactorily conform to specified requirements regarding homogeneity and average soil properties. The fundamental purpose is to ensure that assumptions about soil properties upon which design decisions have been made are in fact met in the constructed facility.

Both homogeneity and average properties are important, although in different ways. Homogeneity is important because marked variations in soil properties within a fill can lead to differential displacements and subsequent cracking or other adverse behavior. Homogeneity is also important because a larger scatter of soil properties means a larger chance that the poorest elements fail to meet minimum specifications. Homogeneity is of particular [illegible] of engineering performance which depend on the weakest link [illegible] example, permeability is important to earthdam performance. [illegible] can be tolerated which has excessive [illegible], due to inadequate compaction, has an excessive potential [illegible].

[illegible] important to ensure adequate average [illegible] adverse performance of a fill. Inadequate [illegible]

Thus, a sampling inspection program has two goals: to control homogeneity and to control average properties. Which, if either, of these goals is more important depends on the specific situation.

Target Properties vs. Sampled Properties

The properties of greatest interest to the engineering performance of an embankment or fill are strength, deformability, and permeability. However, these target properties are cumbersome or expensive to measure directly, so other more easily or quickly measured properties are used in their place. By far the most commonly sampled properties in construction inspection of fills are compaction water content and dry density. The fact that these are correlated to strength, deformability, and permeability makes them useful surrogates.

Tests for Water Content and Dry Density

A variety of tests are available for measuring water content and dry density. Water content can be measured directly by oven drying a specimen and determining change in weight. Dry density can be measured directly by ascertaining the weight and volume of a specimen, as for example, with a sand cone density test.

Water content and dry density can also be measured indirectly using various devices, for example by nuclear gage. These indirect tests are typically less expensive than direct testing but also less accurate. In certain cases economies can be gained by combining a small number of direct tests with a larger number of indirect tests.

Descriptions of field density and water content tests are given in Lambe (1951), Sherard, et al. (1963), Engineer Manual EM1110-2-1911, USBR (1960), and

AASHO and ASTM standard test procedures.

Compaction Specifications

Specifications for compaction quality are most often placed on water content, dry density, or both. For example, water content of the fill at time of compaction might be specified to be within ±2% of standard or modified Proctor optimum. Dry density might be specified to be at least 95% of standard or modified Proctor optimum. These are performance specifications. Specifications are also placed on compaction equipment and procedure. For example, a specified number of passes with equipment of specified minimum capacity may be required in addition to some specified water content range. As an example, on the USAE Carters Dam Project, Georgia, compaction specifications for the imperious core required as-placed water contents to be ±2% standard or modified Proctor optimum and the fill to be compacted by a minimum number of passes using specified equipment. If placed materials were found to have water contents more than ±2% from optimum, the cost fell to the contractor to moisten, dryout, or remove the material. If placed materials were within water content specifications and had been properly compacted, but were less than 95% standard or modified Proctor optimum density, then the cost fell to the owner to undertake additional compaction or to remove the material. These are compliance specifications.

PART V: QUALITY CONTROL CHARTS

Quality control techniques are used by both the contractor and the owner to monitor the progress of construction, and thus to quickly identify changes in soils or operational procedures before these changes adversely affect construction quality.

Quality control differs from acceptance sampling in that QC has the principal purpose of identifying changes in construction materials or procedures before those changes adversely affect construction quality. When a change is detected, efforts are made to find assignable causes and fix them. Acceptance sampling, in constrast, has the principal purpose of assuring that soils placed in a fill meet specifications. Based on acceptance sampling results soils are either accepted or rejected as part of a quality assurance program.

Theory of Control Charts

The main concept underlying control charts is the differentiation of causes of variation in construction quality. Certain variations are simply the result of chance variations in raw materials or in operations about which little is known, such as the uniformity of mixing or the method of construction. Those chance variations occur and their causes may be of which individual are not economically and major fraction of the total variation are due to

many chance causes which produce variation in soil density, moisture content, or other properties are similar to the many forces which cause a tossed coin to land up heads or tails. Such variations follow predictable laws of probability.

On the other hand, other variations in quality are due to assignable causes. In such cases a significant fraction of the variability can be traced to a single cause. By identifying this assignable cause and taking steps to reduce its influence, uniformity of the construction process can be improved. Typical assignable causes include differences among equipment, differences among known materials, differences in operators, sustained changes in weather, or interactions among these factors.

Prediction of Future Variation

If chance variations are ordered in time or space they behave randomly. They show no trends, they show no cycles, and the specific values of any one variation cannot be predicted from surrounding values. On the other hand, the statistical properties of the group of variations can be predicted, provided that

regularity. Therefore, when the process is out-of-control an effort is made to find assignable causes.

Suppose that samples of fixed size n (= number of tests) are taken from lifts being placed in a compacted fill. From each of the n tests a measured value of some soil property results. From these n values certain statistics are calculated, for example the sample mean $m_x=(1/n)\Sigma x_i$, standard deviation $s_x=\sqrt{(1/_{n-1}) \ (n_i-m_x)}$, or range $r_x=(x_{max}-x_{min})$. Being sample results, these statistics will be subject to fluctuation from one sample of n to another. However, if the variations are due to chance causes, the frequency distributions of the sample mean, standard deviation, range, or other statistics are known to follow the regular distributional forms discussed in Part II. For example, the sample mean m_x is known to have a frequency distribution in repeated sampling which is approximately normal (exactly normal if the soil properties being tested are themselves Normally distributed). The average value of the sample mean m_x equals the real mean m_x', while the standard deviation, s_{m_x}, of m_x equals $s_x'/$ n. From Table 4, only 0.2% of the sample means should lie outside a $3s_{m_x}$ interval about m_x'.

Note again, that statistics of the sample data, as for example, the sample mean or sample standard deviation, are denoted here without a prime. For instance, m_x = sample mean. Statistics of the whole sampled population, as for example the actual mean within the entire soil lift, are denoted with a prime.
Thus the s_x' is the actual mean of the sampled population.

Control Charts and Control Limits

A control chart is a device by which the state of statistical control (i.e., that a process is in-control) is operationally defined. It is used to attain control in a new process, and check that control is maintained in an ongoing process.

A control chart is constructed by plotting values of m_X, s_X, r_X or other sample statistics as a function of time or of some other dimension for ordering sample results (e.g., lift sequence number). The sample statistics are plotted against the vertical axis, time or other dimension against the horizontal axis. A horizontal line is drawn through the actual mean m_X', which could be fixed by specification or calculated from data. Two other horizontal lines are drawn, one above m_X' and one below m_X', showing limits which are highly likely to contain the sample results. These are the control limits: the upper control limit (UCL) and the lower control limit (LCL). Fig. 16 shows a typical control chart for individual compaction data.

If sample values are plotted for a substantial range of production and time, and if all these values fall within the interval formed by the UCL and LCL, and if the data show no cycles or runs, then it is concluded that the construction process is in-control for that particular attribute. If the data do not conform to this pattern, then the conclusion is drawn that variability in the constructed product is not explainable by chance factors alone and an assignable cause(s) is sought.

From a statistician's view, control limits are related to the testing of statistical hypotheses. If sample results conform with what would have been predicted by assuming chance causes for variability, then the hypothesis of

random variation is reasonable and accepted. If the results would be improbable based on the hypothesis of random variation, then that hypothesis is rejected. The choice of control limits, and the associated probability of their being exceeded, is arbitrary. For example, the probabililty could be set at 0.01, implying control limits of $\pm$ 1.65 standard deviations; or at 0.002, implying control limits of $\pm$ 3.0 standard deviations. Narrowing the control limits means increasing the risk that the hypothesis of random variation will be rejected when actually it is true. For example, with limits set at $\pm$ 3 deviations there is a chance of 0.002 that a process actually in control will fixed at $(m_X'+3s_m)$ while a lower control limit (LCL) is fixed at $(m_X'-3s_m)$ Usually the standard deviation of m_X, that is, s_{m_X} , is estimated from the data as $s_{m_X}=s_X/\sqrt{n}$. Sometimes s_X itself is specified as a target homogeneity.

Fig 17 shows an m-chart for compaction control data on a dam project.

To control current production, a sample of size n is taken periodically from material placed in the fill and the average m_X of the n test results is plotted on the control chart. If all the m_X lie within the UCL and LCL, the construction process is concluded to be in-control. If any m_X falls outside either the UCL or LCL the process is deemed to be out-of-control. When the deviation outside one of the control limits is adverse, for example, when mean compacted density falls below the LCL, specific cause for the variations are looked for with the intent of improving the construction process and thus the product of that process. When the deviation beyond a control limit occurs on the favorable side, for example, when the mean compacted density exceeds the

UCL, either no action is taken or the causes of this unusually high quality are searched for in order to learn how to permanently improve quality.

Probabilities of individual sample means exceeding the UCL or LCL can be found by reference to Table 4. For soil property data which are themselves Normally distributed the probabilities from Table 4 are exact for $\dfrac{m_x - m_x'}{s_x'}$

For soil property data that are not Normally distributed--presuming that the distributions are not bizarre--the probabilities of Table 4 are still approximately correct even for sample sizes as small as 3.

Control Chart for Sample Range r_x

A control chart on the sample range,

$$r_x = x_{max} - x_{min} \hspace{3cm} -30-$$

shows variation in the range as a function of time. The central line on an R-chart is fixed at the empirical average range in past production, or in special circumstances is set by specification on acceptable variability of the compacted fill. The control limits are usually set at $\pm\ 3s_r$, in which s_r is the standard deviation of the sample range. Both the average range and the standard deviation of the range can also be related to the standard deviation of the soil properties being samples s_x'. Fig. 18 shows an R-chart for compaction control.

If data fall inside the UCL and LCL on an R-chart, the construction process is deemed to be in-control with respect to homogeneity. When a single data point falls outside the UCL or LCL the process is deemed to be out-of-control with respect to homogeneity. In the latter case actions are taken to find assignable causes. A sample result above the UCL is usually considered adverse and efforts should be made to find out the cause of the variability and fix it. A sample result below the LCL's usually considered favorable and efforts can be made to find out what is being done so well so that the construction process can be improved.

Because an m-chart and an R-chart control for different aspects of quality, a process may be in-control on one but out-of-control on the other. An m-chart controls for the mean or average quality of the compacted fill. An R-chart controls for the uniformity with which compacted materials are being placed. Compacted soils may be on average sufficiently dense, but unacceptabely heterogeneous. On the other hand, the soils may be on average sufficiently uniform but unacceptably loose.

Cumulative Reject and Related Charts

Unlike most industrial applications of quality control charts, construction involves a single project with a clearly identified beginning and end. As a result, certain quality control charts are very useful in construction even though they are not widely used in the factory. One of these is the cumulative reject chart.

Cumulative Reject Chart

The cumulative reject chart plots the cumulative numbers of tests having results outside specified limits, against time test sequence number, or a similar indicator of test order. Fig. 19 shows cumulative reject data for a compaction inspection program on the imperious core of a rock fill dam. The upper figure (a) shows cumulative rejects due to inadequate densities. The middle (b) and bottom (c) figures show numerical values of water content and dry density, respectively, for the rejected tests. These are plotted along with the cumulative reject test so that the cause of rejection or any trend in the cause can be readily seen.

In Fig. 19 cumulative reject is plotted against test sequence number. As a result, the slope of the curve gives the rate of rejects at any point during the project. In all, 1175 inspection tests were made on the imperious core, of which 38 were rejected either for being outside ±2% Procter optimum water content or for having dry density less than 95% Procter optimum. The rate of rejects for the entire project is 38/1175 = 3%.

While the average rate of reject tests was 3% for the entire imperious core, during early phases of construction the rate was much higher and during later phases the rate was much lower. At the start of the project the rejection rate reached a high of about 30%, gradually tapering off to a rate of 2% near completion. This gradual but steady decrease in the rate of rejects is reflected in the monotonically increasing cumulative reject chart, and shows a learning or experience effect. In the beginning of the project the contractor and engineers did not have the construction process under

control. The mean quality was low and there was considerable variability in compaction properties. As construction progressed the process was brought into tighter control. The mean quality was better maintained and the variability reduced.

The cumulative reject chart can be used to monitor a number of subtle changes in the construction process. Fig. 20 shows schematically the effect of a major change in the construction process, for example, a change of contractor, change of equipment, or change of borrow material. The change causes a break in the smooth progression of the learning curve, usually starting another learning cycle.

Careful inspection of Fig. 19 shows two such breaks. The first occurs at about test number 50. The reject rate for tests 1 to 50 is about 8%. From test 51 to about 80 the reject rate increases sharply to about 30%. In fact, due to the learning effect the rate of rejects should be expected to decrease not increase as construction proceeds. A retrospective analysis shows that at about test 51 the borrow material was changed. One borrow source was used up to that time, and a second source was used from there on. Because the change was early in the construction process the first learning cycle has barely been put into the program point. Thus, the classic double curve of Fig. 20 is not produced again.

Moving Average, Standard Deviation and Range Charts

Convenient accessories to the cumulative reject chart are the moving average, moving standard deviation and moving range charts. These provide smoothed information on changes in construction output from which trends can be more easily identified.

In a standard m-chart the averages of samples of n tests are plotted as a function of time or some other ordering index. A different set of n tests is used for each point, and the assumption is made that each test is independent of every other. Thus, each plotted sample mean m_x is also independent of the sample means of adjacent to it, presuming that the construction process is in-control and operating in a random manner (Note: in practice the problem of serial correlation in the construction process itself sometimes arises, but such autocorrelation is beyond the scope of the present report). The use of m-charts typically presumes that many data are being collected and that production output is fairly high.

For many cases in construction the rate of testing is more modest or construction proceeds more slowly. Often a considerable time is required to test the individual lifts that are to be tested. In these cases a moving average chart may be more convenient than the standard m-chart. The moving average chart is constructed in the same way as an m-chart, but it provides a better indicator of how the construction process may be changing.

Moving averages are calculated over windows of fixed size n. For a long sequence of data $(x_1, \ldots, x_k)$, an average is calculated for the first n data, then the average is calculated for the next n data

distribution, as illustrated in the above example, may indicate a change in the construction process which needs to be monitored or controlled.

Cumulative Sum (CUSUM) Chart

Changes in the output of a construction process are sometimes more quickly detected by monitoring the change from one test to the next rather than the absolute value of the test. For example the changed conditions which appear in Figs. 19 and 23 become apparent earlier when increments of test results are plotted.

The most common way to monitor increments is by the cumulative sum or cusum chart. The cusum chart uses trends in the QC data to identify process changes, rather than treating the data from each lift by themselves. The major advantage of cusum charts over m- or r-charts (sometimes Shewhart charts after the statistician who first proposed them) are that changes are identified more quickly, particularly modest changes, and that the time or location of the change can be precisely determined.

A cusum chart of a sequence of data $x_1, \ldots, x_i, \ldots$ is constructed by plotting time, location, or sequence number of the data along the horizontal axis, and the cumulative sum $\sum (x_i - m_x')$ along the vertical axis. Fig. 27 shows a cusum chart of the water content data of Figure 24. In practice, the fact that m_x' may not be known precisely is unimportant, as long as the quantity subtracted from the individual measurements is held constant. The reason for subtracting m_x' from the data is only to center the resulting cusum plot on or near the horizontal axis.

As long as the construction process is in control, successive cusum points should lie on a straight line. Some individual x_i will be above the mean m_x', in which case the cusum curve will rise; and some individual x_i will be below m_x', in which case the cusum curve will fall. Thus, an in-control process will wander up and down about the zero axis.

Since m_x' is usually not known in practice, an estimate m replaces the actual value of m_x' in calculating the cusum. That is, the cusum curve plots $\Sigma(x_i - m)$ in which m is a constant roughly equal to m_x'. To the extent that m might be slightly less than m_x', the terms $(x_i - m)$ will, on average, be greater than zero. Thus, the cusum curve will rise at a constant rate; it will be a line of positive slope. To the extent that m might be greater than m_x', the reverse is true and the cusum curve will be a line of negative slope. In all cases, though, the cusum curve of an in-control process will have a linear trend.

When a change occurs in the construction process which alters m_x', the average behavior of the increments $(x_i - m)$ will change. If m_x' drops then, on average the $(x_i - m)$ will go up. Thus, a discrete change in m_x' will cause a discrete break in the linear trend of the cusum curve, after which the cusum data will display a different linear trend. Such breaks are clearly evident in Fig. 27, just preceding the changes previously observed in Figs. 24 and 25. On the cusum chart these changes are well marked.

The procedure for testing statistically whether a change is indeed causing the process to be out-of-control is more involved when using the cusum chart than when using m- or r-charts. The test using a cusum chart can be

Figure 17 -- Shewart control chart (m-chart) for compaction control on a dam
   project.
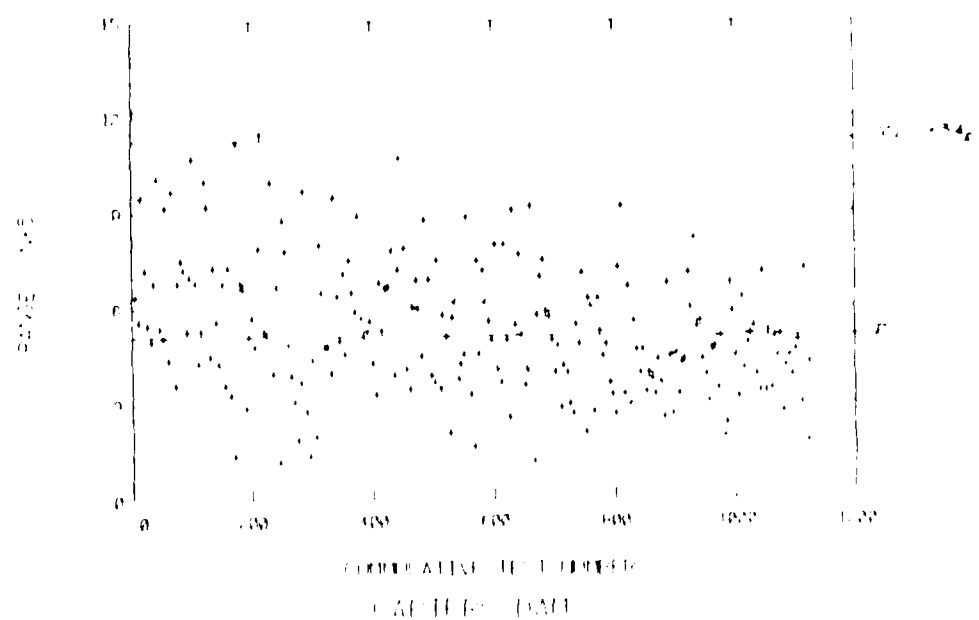
Figure 18 -- Shewart control chart (R-chart) for compaction control on a dam
          project.
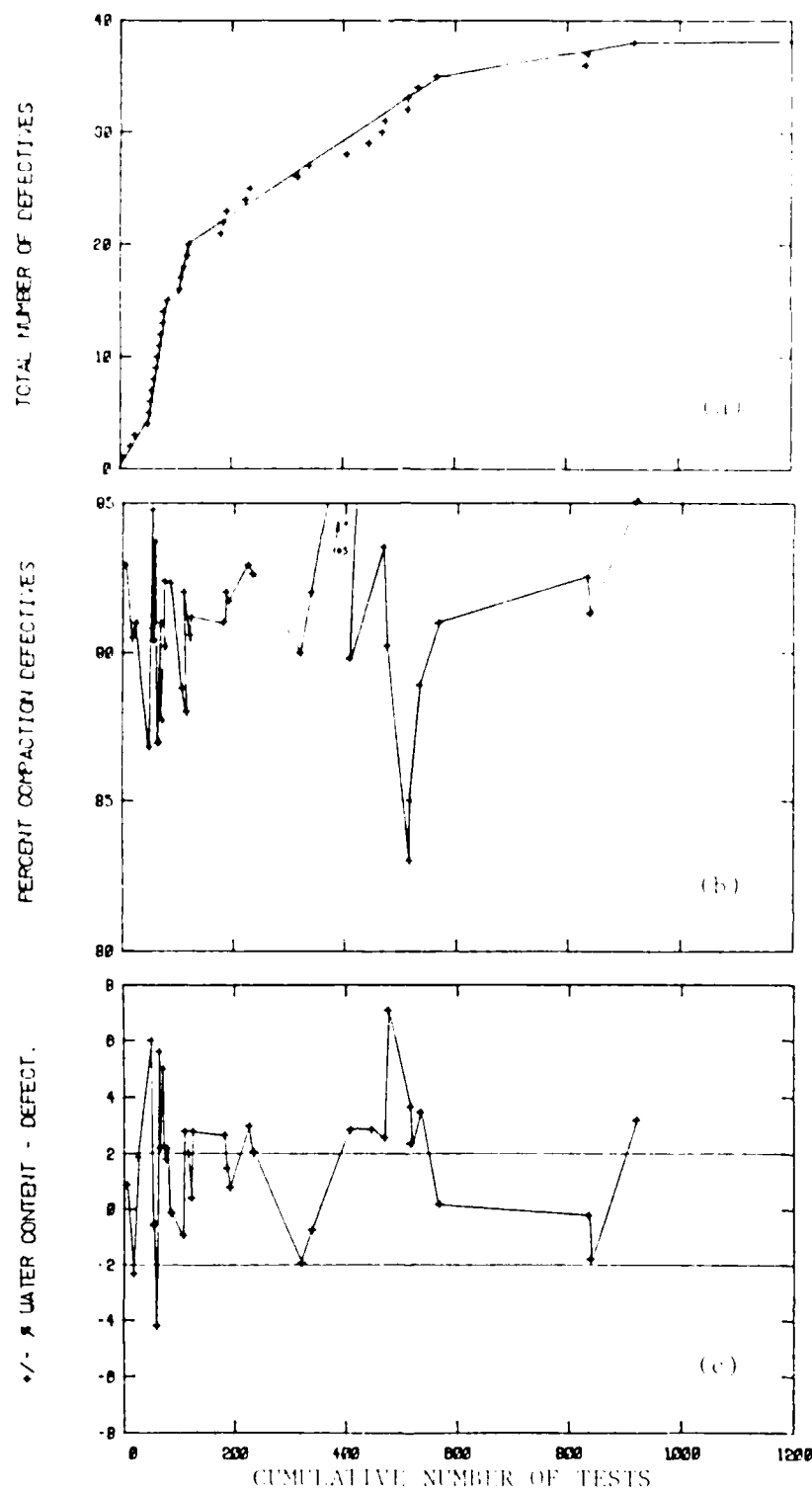
Figure 19 -- Cumulative reject chart for compaction water content and dry
density inspection data from the compacted clay core of a rock
fill dam. Fig. a shows total rejected tests with trend
lines; Fig. b and c show numerical values of rejects
test results.
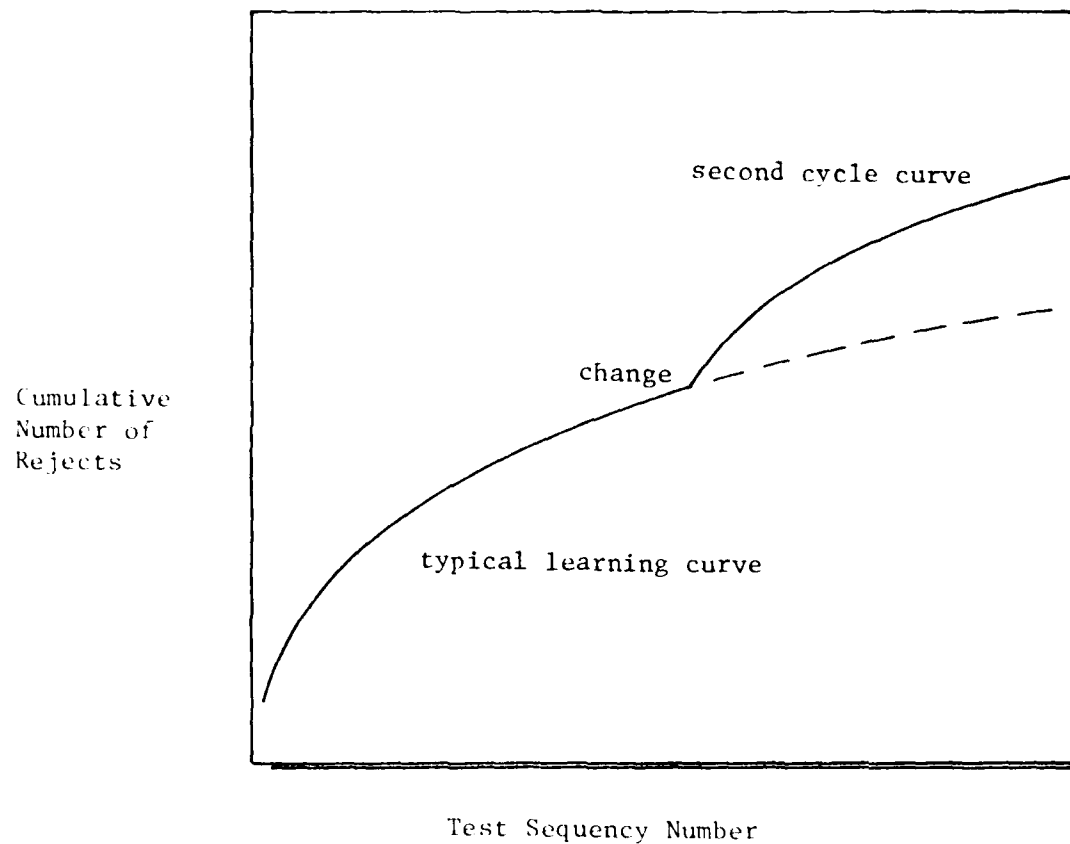
Figure 20 -- Change in construction operation as reflected in cumulative reject chart (schematic).
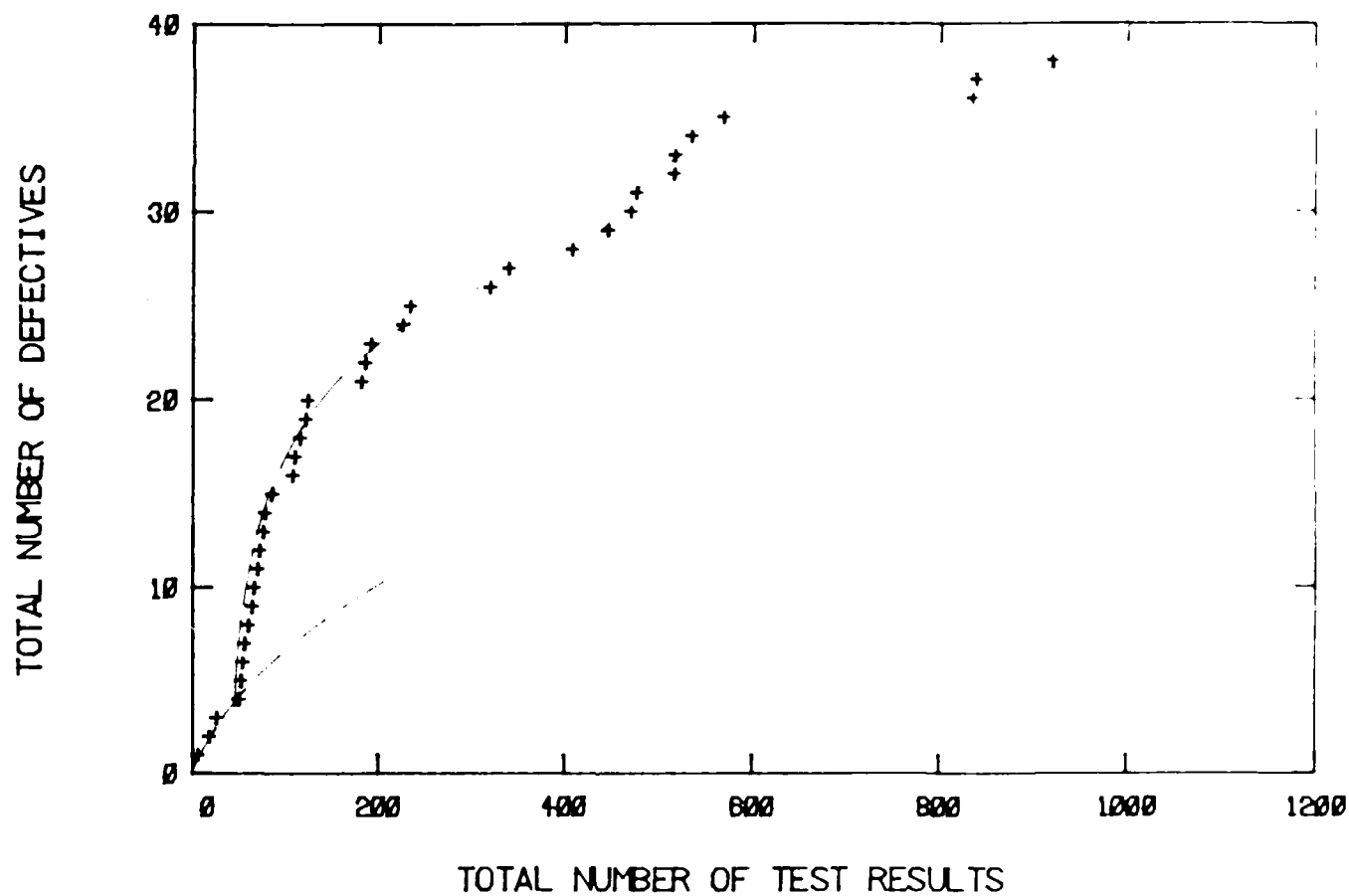
Figure 20b -- Change in construction operations as reflected in an actual cumulative reject chart.
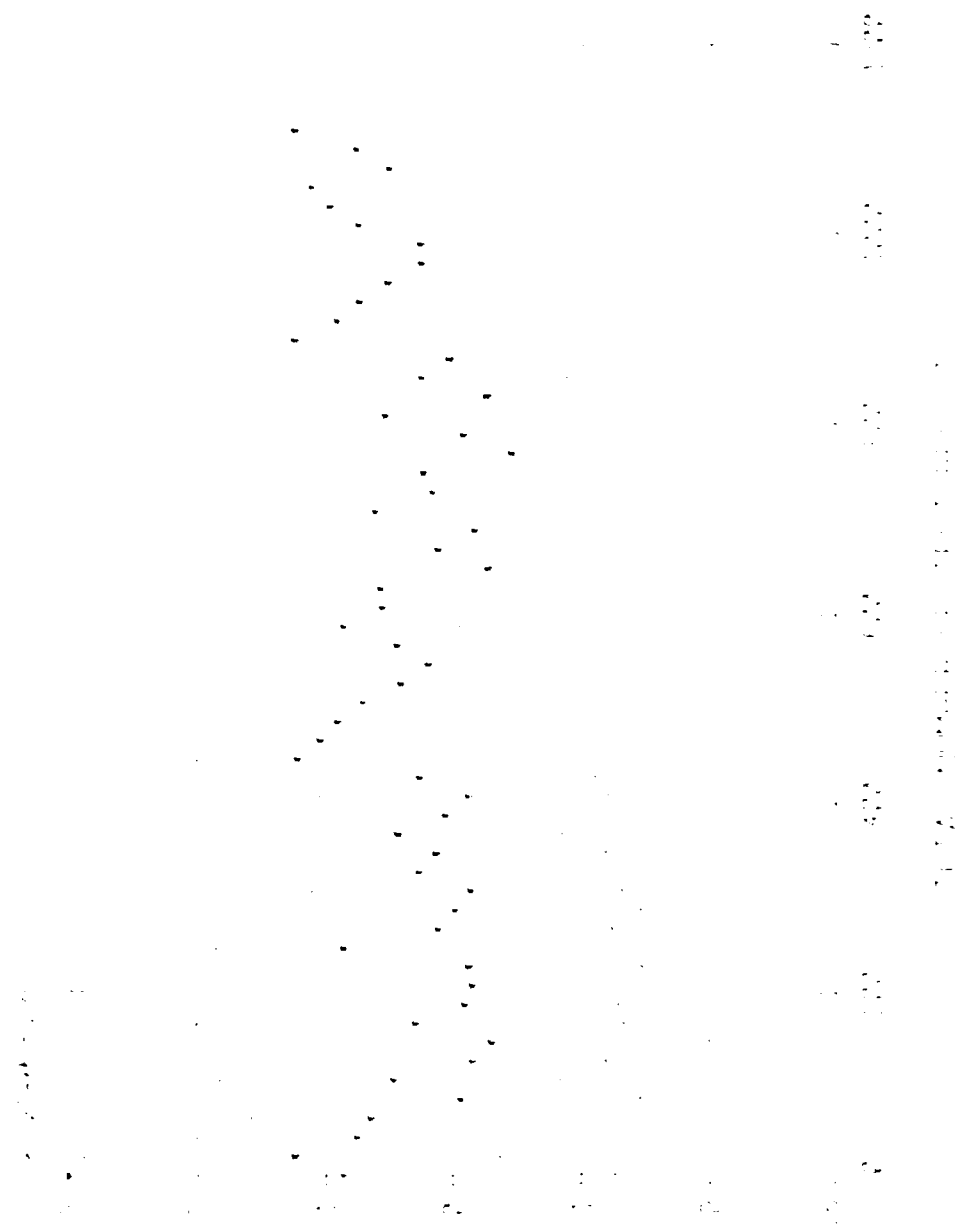
Figure 41. — Horizontal average water content data. Symbol "x" shows mean; bars show +/- standard deviation.
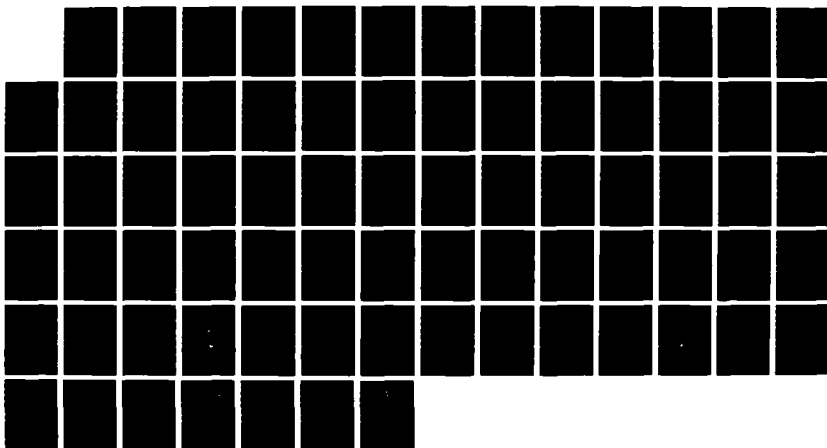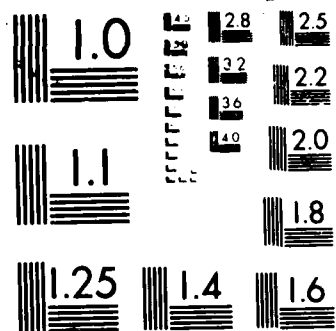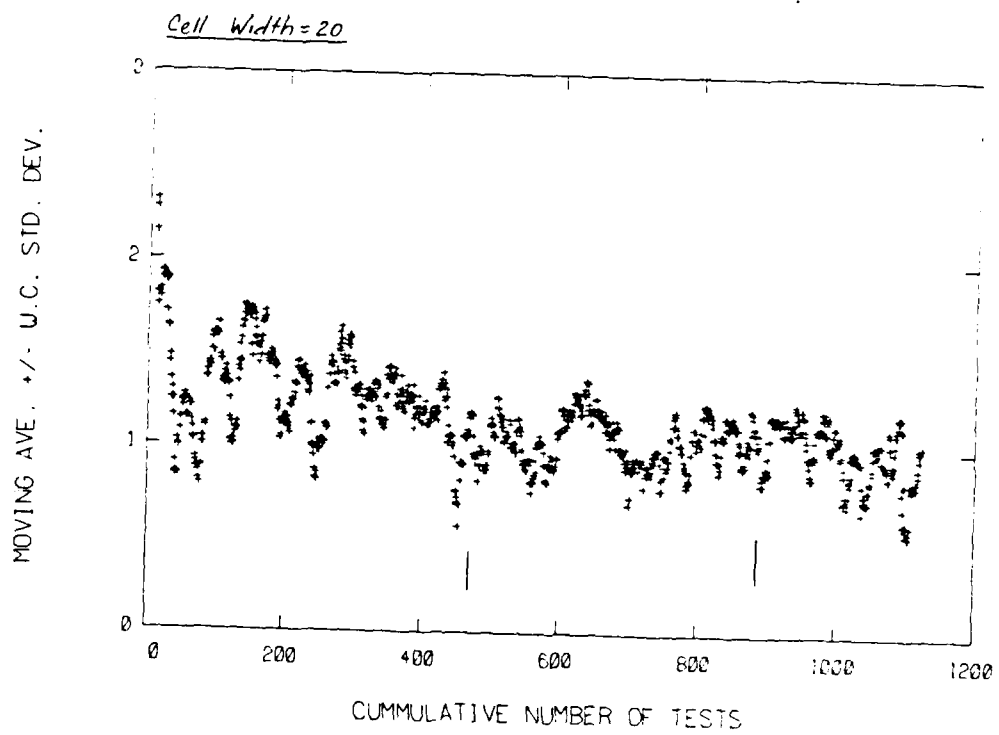
Figure 22 -- Moving standard deviation chart of water content.

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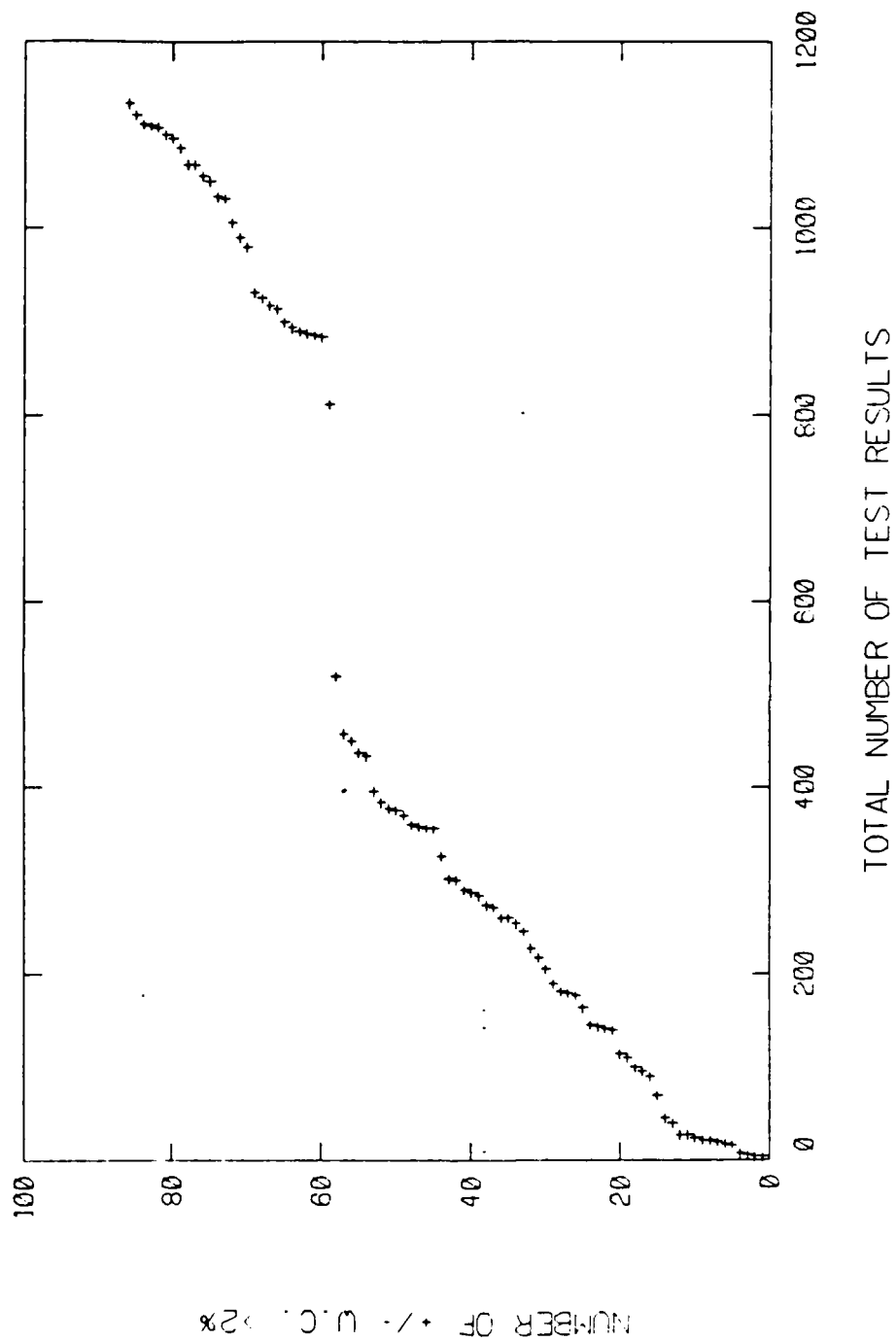
Figure 23 -- Cumulative reject (out-of-specification) chart for compaction water content.
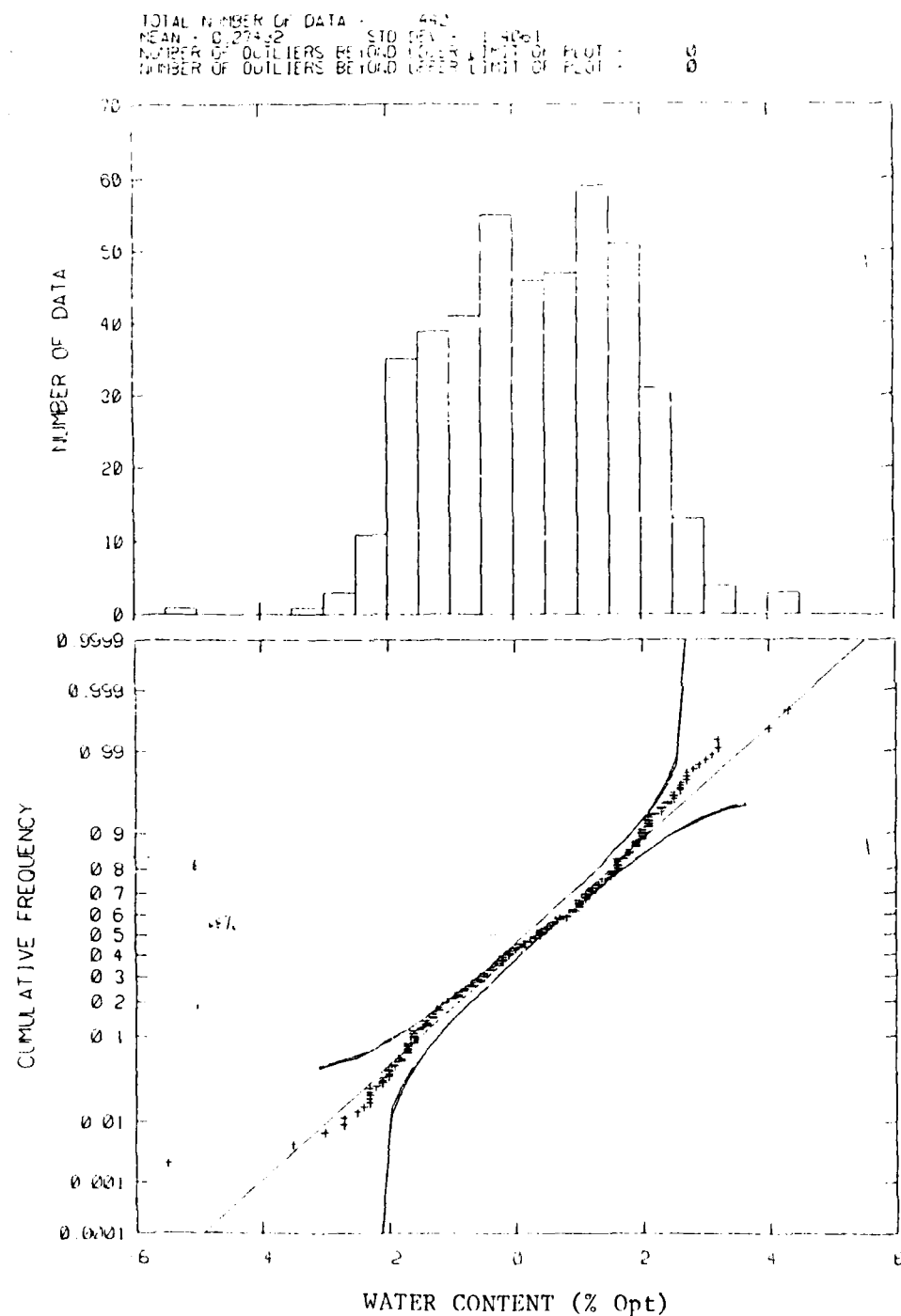
Figure 24 -- Distribution of compaction water content data for tests 1 through 459.

+/- PERCENT WATER CONTENT
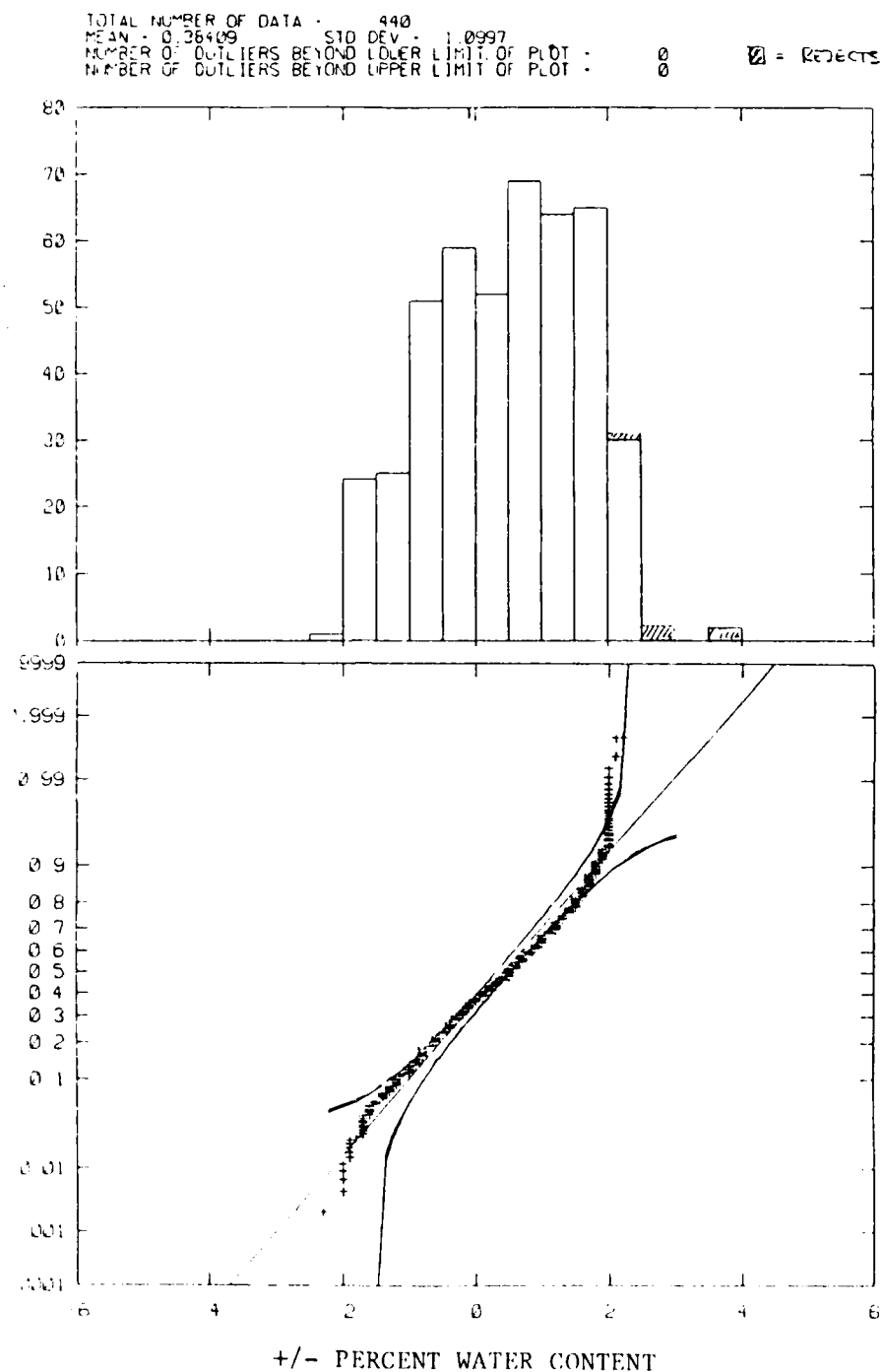
Figure 25 -- Distribution of compaction water content data for tests 460
through 884.

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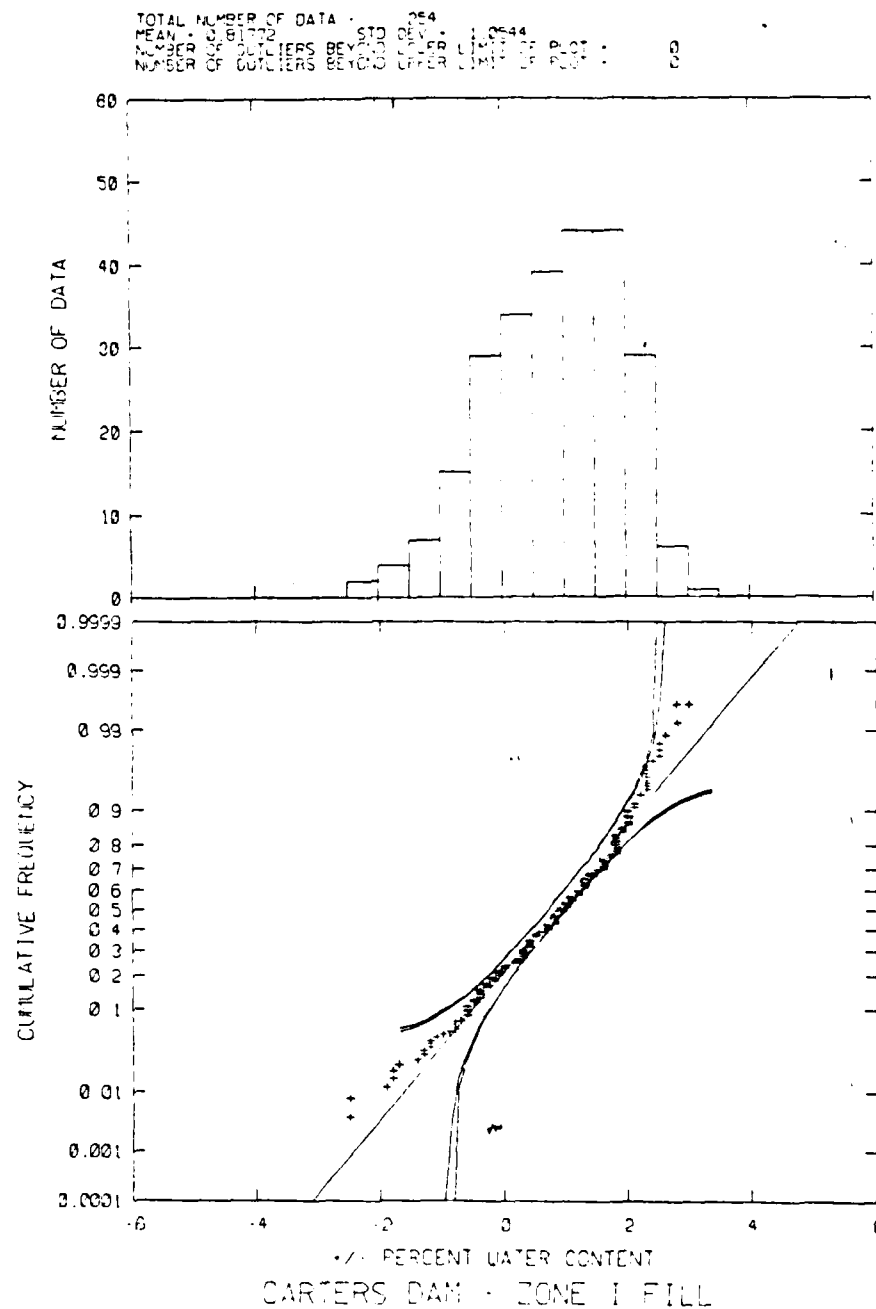
Figure 26 -- Distribution of compaction water content data for tests 885 through 1175.
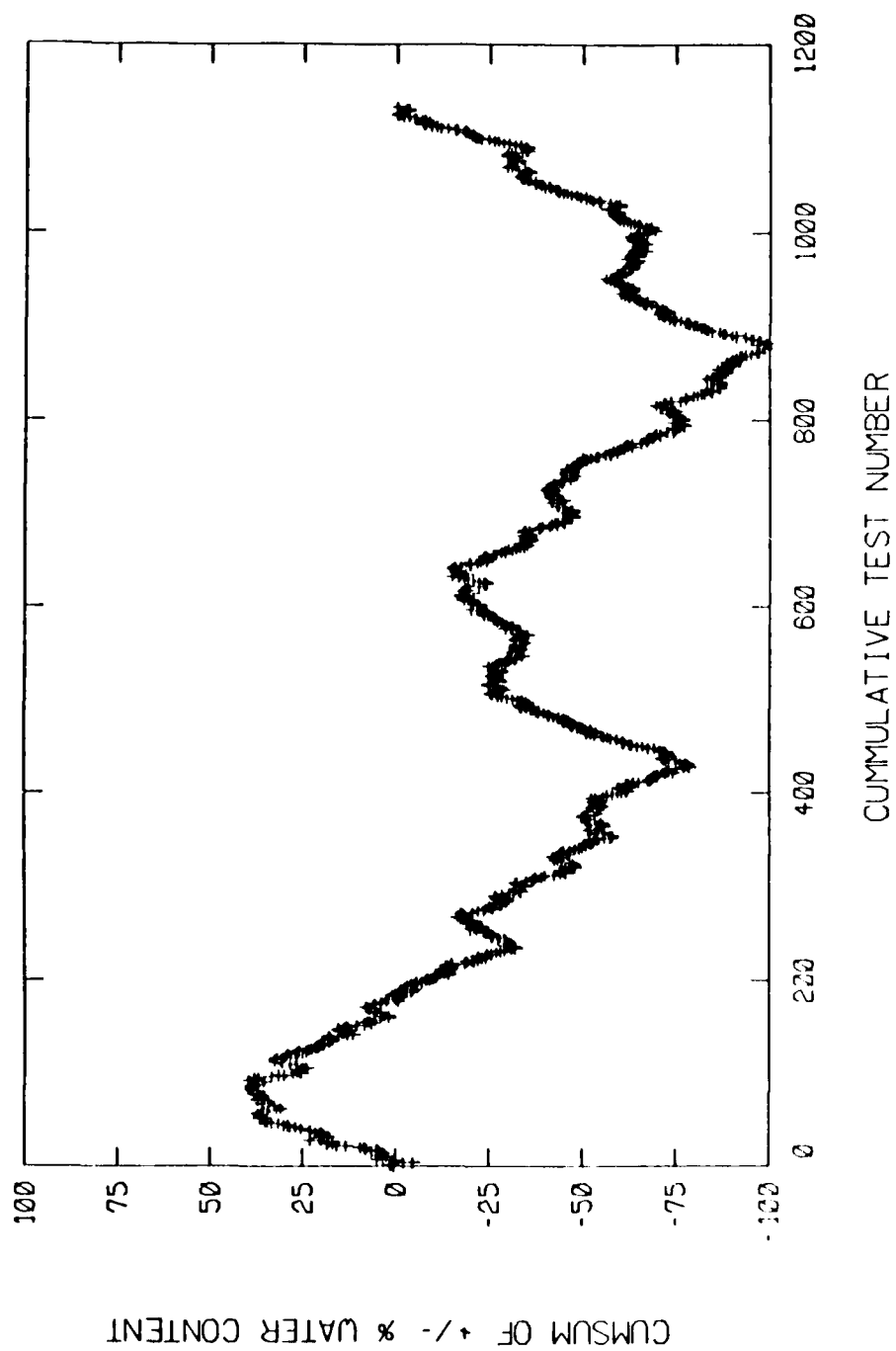
Figure 27 -- Cumulative sum (cusum) chart of the compaction water content data of Fig. 23.

PART VI: QUALITY ASSURANCE BY ACCEPTANCE SAMPLING

The purpose of quality assurance is to test fill as it is placed and to make decisions on whether to accept or reject the fill as conforming to standards. If the fill is rejected, further compaction could be made, the fill could be removed, or some other course of action could be followed pursuant to contractual arrangements between contractor and owner. Acceptance sampling is the quantitative tool used to make the accept/reject decision. The objective of acceptance sampling is not to control quality, but to make decisions.

Structure of An Acceptance Sampling Plan

A simple acceptance sampling plan is structured in the following way:

I.   A random sample of size n is taken from the materials being tested.

II. The results of the n measurements $(x_1, \ldots, x_n)$ are summarized statistically in an index z. For example, z might be the sample average $(1/n) \, x_i$.

III. The index z is compared to a critical value z^*, and if z lies on the correct side of z^* the materials are accepted as satisfactory.

The questions in designing an acceptance sampling plan are how large to make the sample size n, how to summarize the resulting data in an index z, and how to select a critical value z^* such that quality is assured without unduely increasing the cost of construction.

The more stringent the acceptance criteria become, the greater the likelihood of rejecting fill which is in fact satisfactory. The less stringent, the greater the likelihood of accepting fill that is in fact not satisfactory. The problem of acceptance sampling is that, for a given size

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sample, reducing the likelihood of accepting poor material usually means
increasing the likelihood of rejecting good materials, and vice versa. To
simultaneously reduce both the likelihood of accepting poor materials and the
likelihood of rejecting good materials, the sampling plan must be made more
discriminating. This usually increases inspection cost.

Buyer's Risk and Seller's Risk

In specifying an acceptance sampling scheme two risks are balanced,

(a)   The owner's (buyer's) risk of accepting material of poor quality,
      and,

(b)   The contractor's (seller's) risk of rejecting material of good
      quality.

Decreasing one of these risks typically increases the other.

Test results from an acceptance sampling program are variable whether the
fill is truly of acceptable quality or not. Because of this variability, it
may be, for example, that the lowest compaction test results on an acceptable
fill give lower dry densities than the highest test results on an unacceptable
fill.

The top of Figure 28 shows a hypothetical frequency distribution of test
results taken from an acceptable fill. Suppose that the criterion for
accepting the fill as meeting specification is that test results be above γ_d^*.
Because test results are always variable, some fraction of the tests results
will always fall below the acceptance criterion and thus lead to rejection,
even though the fill might in fact be acceptable. This fraction is
proportional to the area under the frequency distribution to the left of γ_d^*.
The probability of the test result lying beneath γ_d^*, and therefore the

probability of improperly rejecting an acceptable fill, is called the

seller's risk.

The bottom of Figure 28 shows a hypothetical frequency distribution of test results taken from an unacceptable fill. Some fraction of these test results will always fall above the acceptance criterion γ_d^* and thus lead to the fill being accepted when in fact it should be rejected. This fraction is proportional to the area under the frequency distribution to the right of γ_d^*. The probability of the test result lying above γ_d^* and therefore leading to acceptance of an unacceptable fill is called the buyer's risk.

For a fixed sampling plan there is an explicit trade off between the buyer's risk and the seller's risk in selecting the acceptance criterion γ_d^*. Higher values of γ_d^* reduce the buyer's risk but raise the seller's risk; lower value of γ_d^* raise the buyer's risk but lower the seller's risk. This trade off can be seen in Fig. 28.

The buyer's risk and the seller's risk can be controlled simultaneously only by making changes in the sampling plan, not just in the acceptance criterion. The purpose of statistical acceptance sampling is to allow the buyer's risk and seller's risk to be quantitatively determined for a given sampling plan and to be appropriately balanced by designing the sampling plan.

Inspecting for Fraction Defective vs. Inspecting for the Mean

Acceptance sampling typically addresses one or both of two aspects of quality:

(a)   The average property of the fill, that is the mean; or,

(b)   The fraction of individual values within a fill which are below some standard, that is, the fraction defective.

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Each aspect of quality may not have the same importance in a particular application. For example, the potential for internal erosion of a fill depends on soil densities at the least compacted places. Conversely, the strength of a fill to resist large slope instabilities more often depends on average soil densities. Acceptance sampling plans differ depending on which aspect of quality is to be assured.

Operating Characteristic Curves

The functional properties of an acceptance sampling plan are usually summarized by an operating characteristic or OC curve. The operating characteristic relates the quality of the fill being sampled--for example its mean density or the fraction of the fill with out-of-specification water content--to the frequency with which the sampling plan leads to a decision to accept. As in Fig. 29, the horizontal axis usually shows the actual fill quality, while the vertical axis shows the probability of acceptance. The Buyer's risk and Seller's risk are read directly from the OC curve corresponding to the definition of good quality and poor quality materials. For example, the probabilities corresponding to the two frequency distributions of Fig. 28 are shown as the Buyer's and Seller's risk, respectively, on Fig. 29. In principle, the better the acceptance sampling plan, the steeper the OC curve in the vicinity of the contract-specified quality of the fill. A steep OC curve reduces both the Buyer's risk and the Seller's risk.

The shape of the OC curve depends on the design of the acceptance sampling plan, and can be used to make economic decisions about the reasonable extent and cost of sampling. Usually, the easiest way to steepen the OC is by

increasing the sample size and thus sampling cost. The remainder of Part VI
discusses the relation between a sampling plan and its corresponding operating
characteristic, and how a sampling plan can be designed to achieve a desired
OC curve.

Acceptance Sampling to Give Assurance on the Mean

This section considers acceptance sampling plans the intent of which is to
assure that average properties of placed materials meet specification. Two
types of specification are considered, single limits and double limits. Using
single limits the concern is that the average properties are, for example,
greater than some specified value. For instance, average compacted dry density
is to be greater than 95% standard or modified Proctor optimum. Using double
limits the concern is that the average properties are between two values. For
example, average compaction water content is to be within ±2% standard or
modified Procter optimum.

The sub-sections first consider the case of known or specified material
variability, that is, known standard deviation. This case is mathematically
easier than the more general case of unknown variability, and does sometimes
occur in practice. The more general case of unknown variability is treated
afterward.

Single Limit, Standard Deviation Known

Suppose specifications call for soil with an average or mean compacted dry
density of $m_x' = 120$ pcf. Suppose also that the dry density of the compacted
fill is known to have a constant standard deviation of $s_x' = 15$ pcf. An

acceptance sampling plan to give assurance regarding the mean is constructed

such that material actually having a mean of at least 120 pcf (i.e., good

material) will be rejected no more frequently than some fixed value α. As

before, α is the Seller's risk. Simultaneously the sampling plan is

constructed such that material whose mean is substantially less than 120 pcf

(i.e., poor material) will be accepted no more frequently than some other fixed

value β. As before, β is the Buyer's risk. For the acceptance sampling plan

to be operational, a specific definition of what is meant by "substantially

less" must be adopted. In Fig. 29, poor material is defined as being an

average density less than 110 pcf.

The procedure for acceptance sampling with one fixed limit on the mean is

the following

1. Take a random sample of n tests

2. From the results $x_1 \ldots x_n$ calculate the mean

   $$m_x = (1/n) \Sigma x_i.$$

3. Compare m_x with a specified acceptance value m^*;

   if $m \geqslant m^*$, then accept
   if $m < m^*$, then reject.

The OC curve for a sampling plan regarding the mean shows the probability

of acceptance as a function of the true mean value of the material. $M_x{}'$, as in

Fig. 29. The OC curve is constructed by using the standardized variable Z_m,

$$z_m = \frac{m^* - m_x{}'}{s_x{}' / \sqrt{n}} . \tag{-31-}$$

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The denomination in Eqn. 31 is the standard deviation of the sample mean $(1/n) \Sigma x_i$ over repeated samples (cf., Eqn. 25). That is, the denomination expresses the variability one naturally expects among different sets of tests. The numerator is the separation between the acceptance criterion in * and the true average quality of the soil $m_x{}'$. The variable Z_m is the number of standard deviation separating m^* for $m_x{}'$, and thus can be used to calculate the fraction of samples in which the deviation of the sample mean for $m_x{}'$ is given than $m^*-m_x{}'$.

When the property being tested has a Normal frequency distribution, the frequency distribution of z_m over multiple samples is exactly Normal. Yet, even when the property being tested does not have a Normal frequency distribution, the frequency distribution of z_m is still approximately Normal. The probability of accepting material with actual mean $m_x{}'$ is found by comparing z_m with Table 4 to find the corresponding frequency with which a standard Normal variable exceeds z_m.

Consider a sampling plan with an acceptance mean $m^*=105$ pcf and sample size n=6. Under this plan, n=6 tests are made, the mean m_x of the results is calculated, and if $m_x > 105$ pcf the material is accepted. If $m_x < 105$ pcf the material is rejected.

The OC curve for this plan is calculated by computing the quantity z_m in Eqn. 31 and looking in Table 4 to find the probability of a standard Normal variable having an absolute value larger than Z_m. Because the standard Normal distribution is symmetric about Z=0, the area under the distribution above +Z is the same as the area under the curve below -Z. For example, if the true

mean were $m_x' = 120$ pcf and $s_x' = 15$ pcf, then $z_m = (105$ pcf -120 pcf$)/(15$ pcf$/\sqrt{6}) = -2.4$. Thus, the probability of accepting good material with the specified mean density 120 pcf equals the probability of a standard Normal variable being algebraically greater than -2.4, that is, about 0.01. Other points on an OC curve such as that in Fig. 29 are evaluated by substituting corresponding values of m_x' into Eqn. 31.

An acceptance sampling plan with regard to the mean is designed by specifying a Seller's risk α and a Buyer's risk β. The Seller's risk is the probability of rejecting fill which in fact is of better quality than some decided upon acceptable quality level (AQL), or "good" material. The Buyer's risk is the probability of accepting fill which is in fact of poorer quality than some decided upon unacceptable quality level (UQL), or "poor" material. The AQL and UQL are engineering decisions and must be quantitatively specified to give meaning to the notions of good and poor quality material. The sampling plan is defined by a sample size n and an acceptance level m^*. The procedure to find (n, m^*) is:

1. Specify

   α = Seller's risk
   β = Buyer's risk
   m_a = Acceptable quality level of mean (AQL)
   m_u = Unacceptable quality level of mean (UQL)

2. Find standard Normal variables (Table 4) with frequencies of not being exceeded equal to $(1-\alpha)$ and β,

   $z_{1-\alpha}$ = standard Normal variable with frequency of not being exceeded $(1-\alpha)$.

   z_{β} = standard Normal variable with frequency of not being exceeded β.

3. Write the two equations

$$\frac{m^* - m_a}{s_x'/\sqrt{n}} = -z_{1-\alpha} \rightarrow \text{Sets Seller's risk} \qquad -32-$$

$$\frac{m^* - m_u'}{s_x'/\sqrt{n}} = -z_\beta \rightarrow \text{Sets Buyer's risk} \qquad -33-$$

4. Solve simultaneously for n and m*.

An example is shown in Plate 3 and Figure 29.

Are Compaction Data Normally Distributed?

Experience has shown that empirical data on water content and dry density for compacted soils are often well approximated by Normal distributions. Examples are shown in Fig. 30. Specific experimental data may on occasion be better fit by distributions other than the Normal, but this is uncommon.

Actually, the empirical fact that the variability of soil properties is often well approximated by the Normal frequency distribution is not surprising. The Central Limit Theorem, one of the cornerstone of statistics (Benjamin and Cornell, 1970), shows that when variability among data is caused by the cumulative effect of a large number of small pertubations or errors, the resulting frequencies of observations should exhibit a Normal distribution.

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Presumably for this reason, Normal distributions are common across the broad spectrum of experimental science. In Part V of this report, deviations of observed frequency distributions from Normality were used to identify changes in construction process and inspection procedures.

Single Limit, Standard Deviation Unknown

The development of an acceptance sampling plan to assure the mean when the standard deviation is unknown is similar to the case when the standard deviation is known, except that the index z_m involving the known standard deviation s_x' is replaced by an index t involving the sample standard deviation s_x.

The inspection sampling procedure is

1. Take a random sample of n tests.

2. Calculate the mean and standard deviation of the test results,

$$m_x = (1/n) \; \Sigma x_i \qquad\qquad -34-$$

$$s_x = (1/n-1) \; \Sigma (x_i - m_x)^2 \qquad\qquad -35-$$

3. Evaluate the sample statistic

$$t = \frac{m_a - m_x}{s_x/\sqrt{n}} \qquad\qquad -36-$$

   in which m_a = AQL.

4. Fix an acceptance criterion t*;

   if t > t* then accept,
   if t < t* then reject.

The OC curve for this plan is a function of both the actual mean and actual standard deviation. The vertical axis of the OC curve is the probability of accepting the tested material. The horizontal axis is the non-dimensional quantity $\lambda = (m_a' - m_x')/s_x'$, which involves the specified AQL and both the real mean and the real standard deviation cf., plot in S.W..

The OC curve is calculated in a manner similar to that when the standard deviation is known, but whereas the variation of z_m across different samples can be approximated by a Normal frequency distribution, the variation of t across different samples -- at least for small n (Say n > 20) -- is wider than for z_m and must be approximated by a so-called student -t frequency distribution. The variation in t is wider than in z_m because the sample standard deviation varies somewhat from sample to sample. As n gets larger, the variation of s_x about sx' becomes smaller, and the student t distribution approaches a Normal distribution.

To design an acceptance sampling plan for the mean when the standard deviation is unknown the procedure is:

1. Specify,

α = Seller's risk
β = Buyer's risk
m_a = Acceptable (mean) Quality Level (AQL)
m_u = Unacceptable (mean) Quality Level (UQL)

2. Make a rough estimate of s_x'

3. Compute $\lambda = \dfrac{m_a - m_u}{s_x}$.   -38-

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This is the value of λ when the actual mean m_x' equals the UQL m_u'.

4. Make a rough estimate of n from Fig. 32.
   Material having the value of λ from step 3 should be accepted only β fraction of the time. Find n in Fig. 32 providing β probability of accepting material of quality λ

5. Find the acceptance criterion t* corresponding to a frequency of not being exceeded $(1-\alpha)$ from Table 6, using ν = n-1 degrees of freedom.

6. Specify sampling plan by

   n = sample size
   t* = acceptance criterion

$$t = \frac{m_x - m_a}{s_x/\sqrt{n}} = \text{test statistic.} \qquad -39-$$

Plate 4 shows the design of a sampling plan for the same condition as in Plate 3, but that the standard deviation is not known. The effect of not knowing the standard deviation in this case is that the sample size must be increased by one test, from 9 to 10, to obtain the same precision in the OC curve.

Double Specification Limits, Standard Deviation Known

Certain material properties, as for example compaction water content, have specification limits both above and below their target value. Soil moisture should be within some ± interval of optimum, say, no wetter than +2% of standard or modified optimum Proctor and no dryer than -2%. An acceptance sampling plan with double specification limits intends to assure that a

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material property is within the defined interval.

An acceptance sampling plan with double limits is designed by specifying two acceptance bounds. If the sample average lies between these bounds, the lift is accepted. If the sample average lies outside, the lift is rejected. The bounds are chosen to conform to specific values of the Buyer's risk and the Seller's risk.

If the variability of the soil properties as measured by the standard deviation is known, then the variability of the sample average of n tests from one sample to another is also known (i.e., $s_m = s_x'/\sqrt{n}$). As before, if the soil properties are assumed to have a Normal frequency distribution, the variation of the sample average also has a Normal distribution. Even if the soil properties are not Normally distributed, the distribution of the sample average is usually still approximately Normal.

Let the target value or acceptable quality level of the average soil properties be m_a. If indeed the average soil property is m_a, the sample average of n test results will vary about m_a as shown in Fig. 33. This sample-to-sample variability of m_x is centered on m_a and has standard deviation $s_x/\sqrt{n}$.

Let $m_U{}^*$ and $m_L{}^*$ be the upper and lower acceptance limits on the sample mean m_x. If m_x is greater than $m_U{}^*$ or less than $m_L{}^*$ the lift is rejected. The Seller's risk α is the frequency with which the sample mean m_x lies outside $m_U{}^*$ and $m_L{}^*$ when in fact the true mean is m_a. That is, the Seller's risk is the shaded areas in under the frequency distribution of m_x in Fig. 33. Each tail area has frequency (i.e., probability) $\alpha/2$.

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Let UQL_U and UQL_L be the upper and lower unacceptable quality levels. If the actual average soil property lies just outside the UQL_U or UQL_L, there is still a chance that sample variability will allow the measured sample mean to lie inside the range (m_L^*, m_U^*), and thus lead to the lift being improperly accepted. This frequency is the Buyer's risk β. The sampling variability of m_X for two lifts which have true means equal to UQL_U and UQL_L are shown in Fig. 34. The frequency (i.e., probability) with which the sample mean from these soils lies within the interval (m_L^*, m_U^*) is shown by the shaded areas under the respective frequency distributions. Each tail area equals β, the Buyer's risk.

To design an acceptance sampling plan on the mean with double specification limits, two constraints must be satisfied, the Seller's risk and the Buyer's risk. Two parameters can be adjusted, the sample size n and the location of the acceptance limits m_U^* and m_L^*. The sample size controls the width of the frequency distribution of m_X, in that the standard deviation of m_X equals $s_X'/\sqrt{n}$; while the limits m_U^* and m_L^* control where the frequency distributions are cutoff to yield α and β.

From Table 4, the tail area under a Normal frequency distribution can be related to numbers of standard deviation on either side of the mean. Let z_p be the number of standard deviations below which the area under the Normal frequency distribution is $(1-p)$ (i.e., z_p is the standard Normal variable which has probability p of not being exceeded). For example, from Table 4, $z_{0.975} = + 1.96$, and $z_{0.025} = -1.96$. Then, Figs. 33 and 34 lead to four relationships from which an acceptance sampling plan can be designed:

$$\frac{m^*_U - m_a}{s'_x /\sqrt{\bar{n}}} = z_{1-\alpha/2} \qquad -40-$$

$$\frac{m^*_L - m_a}{s'_x /\sqrt{n}} = z_{\alpha/2} \qquad -41-$$

$$\frac{m^*_U - UQL_U}{s'_x /\sqrt{n}} = z_{\beta} \qquad -42-$$

$$\frac{m^*_L - UQL_L}{s'_x /\sqrt{n}} = z_{1-\beta} \qquad -43-$$

As an example, consider an inspection plan for compaction water content in which the Seller's risk and Buyer's risk were set at $\alpha=0.05$ and $\beta=0.10$, respectively. The target value of average water content is Proctor optimum, and intolerable deviation from the target has been decided to be ±3% water content. Assume that from project records the standard deviation were known to be about 1.5%. For these conditions, Eqns. 40 to 43 become

$$\frac{m_U^* - 0}{1.5/\sqrt{n}} = 1.96 \qquad\qquad -44-$$

$$\frac{m_L^* - 0}{1.5/\sqrt{n}} = -1.96 \qquad\qquad -45-$$

$$\frac{m_U^* - 3\%}{1.5/\sqrt{n}} = -1.282 \qquad\qquad -46-$$

$$\frac{m_L^* - (-3\%)}{1.5/\sqrt{n}} = 1.282 \qquad\qquad -47-$$

Solving the first two equations simultaneously gives,

$$m_U^* = -m_L^* . \qquad\qquad -48-$$

Solving the first and third equation simultaneously gives,

$$n = [(\frac{1}{3})\ (1.5)\ (1.96 + 1.282)]^2 = 2.62, \qquad\qquad -49-$$

or rounding off, n=3. Putting n=3 into the equation for Seller's risk gives $m_u^* = 1.7\%$, $m_L = -1.7\%$. Putting n=3 into the equation for Buyers risk gives $m_u^* = 1.9\%$ and $m_L = -1.9\%$. Choosing ±1.8% as the acceptance limits gives a Seller's risk of α=0.38 (i.e., less than 5%) and a Buyer's risk of β=0.08

(i.e., less than 10%). The OC curve for this plan is shown in Fig. 35. Another example is given in Plate 5.

Double Specification Limits, Standard Deviation Unknown

When the standard deviation is unknown the procedure for specifying an acceptance sampling plan is much the same as when the standard deviation is known, except that the sample standard deviation s_x replaces the known standard deviation s_x' in Eqns. 40 to 43, and the Student-t distribution (Table 6) replaces the Normal distribution (Table 4).

As for the case of a single specification limit, the test statistic is,

$$t = \frac{m_x - m_a}{s_x/\sqrt{n}} \; . \qquad\qquad -50-$$

For sample sizes above about n=20 these modifications are unnecessary because the sample standard deviation s_x is sufficiently close to the actual standard deviation s_x'.

The inspection sampling procedure is,

1. Take a sample of size n

2. Calculate the mean and standard deviation of the test results,

$$m_x = (1/n)\sum x_i \qquad\qquad -51-$$

$$s_x = (1/n-1)\sum (x_i - m_x)^2 \qquad\qquad -52-$$

3. Evaluate the sample statistic

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$$t = \frac{m_x - m_a}{s_x/\sqrt{n}}$$

-53-

4. Fix an acceptance criterion t*;

   if $|t|$ < t* then accept
   if $|t|$ > t* then reject.

As in the case of a single specification limit, an inspection sampling plan

with double limits is designed by specifying a sample size n and a criterion

t*. An initial guess at $s_x{}'$ is made, and Fig. 32 is used to estimate a sample

size n based on the quantity,

$$\lambda = \frac{|UQL - m_a|}{s_x}$$

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in which m_a is the target soil property and UQL is either the upper or lower

unacceptable quality level. This assumes that UQL_U and UQL_L are symmetrically

placed about the target m_a. See Duncan (1974) or Grant and Leavenworth (1972)

for asymmetric cases. Unacceptable materials at either the UQL_U or UQL_L should

be accepted only with frequency β. Thus, knowing the Buyer's risk β and the

number of standard deviations λ separating the UQL from m_a, an initial sample

size can be chosen from Fig. 32. Using UQL=$\pm$3%, m_a=0%, and $s_x \cong 1.5$%, as

before, Fig. 32 leads to n $\cong$ 4.

The acceptance criterion t* is found from a table of the Student's-t

frequency distribution (Table 6). This table provides the frequencies with

which given values of the test statistic of Eqn. 50 are exceeded due to random

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sampling variations when in fact the soils being inspected are of target quality m_a. Because both unacceptably high and unacceptably low values will be rejected, the Seller's risk is the sum of the frequencies with which the test index of Eqn. 50 lies above +t* and below -t*. Thus, t* is set so that the tail areas on either side each have probability $\alpha/2$. For the Student's-t frequency distribution these tail areas depend on the sample size taken, though the so-called degrees of freedom $\nu=n-1$. For these conditions, Table 6 gives a t* value of 3.25.

5.   <u>Specify Sampling Plan:</u>

     a)   Take sample of 4 tests
     b)   Calculate sample mean m_x and sample standard deviation s_x
     c)   Calculate the test index

$$t = \frac{m_x - 0}{s_x/\sqrt{4}} \qquad\qquad -55-$$

     d)   If -3.18 < t < +3.18, then accept lift.
           If t < -3.18 or t > +3.18, then reject.

<u>Acceptance Sampling for Fraction Defective</u>

The following section considers the case in which an inspection sampling plan is employed to assure that the fraction of out-of-specification material in a compacted fill is within tolerable limits. Such plans are generally called acceptance sampling for fraction defective.

If the soils data display a Normal or bell-shaped frequency distribution, there is an exact mathematical relationship between the mean and standard deviation of the data on the one hand, and the fraction defective on the other. This is shown schematically in Fig. 36.

Setting the lower limit of acceptable values or the UQL at

L = lower limit of acceptability

a standardized deviate z_L is defined as the number of standard deviations s_x' separating the mean of the data m_x from the lower limit L,

$$z_L = \frac{m_x' - L}{s_x'} \qquad\qquad -56-$$

If the data are normally distributed, z_L is uniquely related to the fraction defective, as shown in Fig. 36. Numerical values of this relationship are found in Table 4 or can be approximated by Eqn. 24. Fig. 36 illustrates that the higher the mean and the lower the standard deviation, the lower the fraction defective. An acceptance sampling plan for fraction defective is structured in the following way:

1. Test a random sample of size n to obtain the data $x_1, \ldots, x_n$.
2. From the results, calculate a sample mean m_x, sample standard deviation s_x, and test index

$$z = \frac{m_x - L}{s_x} . \qquad\qquad -57-$$

Depending on the specific problem, the formula for z may vary somewhat.

3. Compare the computed value of z with a critical value z*:

If $z > z^*$, then accept.
If $z < z^*$, then reject.

The choice of n and z^* defines the performance properties of the sampling plan. These parameters are usually chosen to satisfy specified levels of Buyer's risk and Seller's risk.

Operating Characteristic for Fraction Defective Sampling

The operating characteristic or OC curve summarizes the discriminatory power of an acceptance sampling plan. The OC curve shows how the probability of accepting a lift or other quantity of material varies as a function of the quality of the material being inspected. For plans aimed at fraction defective the OC curve relates probability of acceptance to the fraction defective in the lift.

Consider an acceptance sampling plan for percent compaction specified by:

n = 5
L = 95% maximum Proctor density
z^* = 1.645 (i.e., 5% of the soil less than 95% max. density).

Presume the standard deviation is known to be $s_x' = 2\%$. Under this plan 5 tests are made. The average of the tests m_x is compared to L through $z = (m_x - L)/s_x'$. If $z > 1.645$ the material is accepted; if $z < 1.645$ the material is rejected. The OC curve for this plan relates the probability of accepting the material to the actual fraction of the lift compacted to less than 95% Procter maximum.

For Normally distributed material with known standard deviation there is a unique relation between the fraction defective and the mean. For $s_x' = 2\%$ and L = 95% Table 4 is used to find the following relations:

Fraction defective	p'	0.05	0.10	0.15	0.20	0.25
Mean	m_x'	98.3	97.6	97.1	96.7	96.4

Thus, the horizontal axis of the OC curve can be expressed either as actual fraction defective or as actual mean.

For a given fraction defective or given mean, the probability of accepting the material equals the probability that the test result z is greater than $z^* = 1.645$. This probability can be determined by noting that z is itself Normally distributed. With L and s_x' fixed, z depends only on the mean m_x of the test results. When sampling from a Normally distributed population, the frequency distribution of the sample mean is also Normal (Part II). Thus, the probability that z>1.645 is found by calculating the mean and standard deviation of z and referring to Table 4.

The mean and standard deviation of z are found by the method described in Part II,

$$m_z = \frac{m_x' - L}{s_x'} \qquad\qquad -58-$$

$$s_z = \frac{s_m}{s_x'} = \frac{s_x'/\sqrt{n}}{s_x'} = 1/\sqrt{n} \qquad\qquad -59-$$

Table 4 is entered by calculating the number of standard deviation of z separating m_z from the acceptance criterion $z^*=1.645$. The corresponding number on the vertical axis is the probability of rejection (i.e., the tail area of the Normal curve, or $\Pr\{z<z^*\}$). The probability of accepting the material is the complement of this number,

$$\Pr\{\text{accepting}\} = 1 - \Pr\{\text{rejecting}\}. \qquad -60-$$

This procedure is illustrated in Plate 6.

The entire OC curve is found by calculating the probability of accepting the material for various values of actual fraction defective. For the sampling plan above, the full OC curve is shown in Fig. 37. If none of the material is defective the probability of accepting is 1.0, and as the actual fraction defective increases (i.e., as the mean of the material decreases) the probability of accepting goes down.

Single Limit with Known Standard Deviation

The main question in designing an acceptance sampling plan is to decide upon a sample size n and an acceptance criterion z^*. These choices dictate how the plan performs with respect to Buyer's risk and Seller's risk. Let the probability of improperly accepting unsatisfactory material, the Buyer's risk be,

$$\text{Buyer's risk} = \beta; \qquad -61-$$

let the probability of improperly rejecting satisfactory material, the Seller's risk be,

An OC curve is defined by specifying two points through which it passes. For this purpose the Buyer must specify a maximum fraction defective that he considers tolerable and which would be accepted under the plan only some fraction β of the time. This poor quality material as a fraction defective is denoted p_u'. At the same time, a target or desired quality level is specified which would be accepted at least $(1-\alpha)$ fraction of the time. This good (i.e., acceptable) quality as a fraction defective is denoted p_a'. The OC curve can be made to pass through the two points $(p_a', 1-\alpha)$ and (p_u', β) by adjusting the sample size n and acceptance criterion z^*.

For example, consider that acceptable material has $p_a' = 0.01$ fraction defective and an unacceptable material has $p_u' = 0.10$ fraction defective. To fix the two points of the OC curve specified by the Buyer's risk and the Seller's risk, the first task is to calculate the corresponding averages m_a' and m_u' which would give fractions defective of $p_a' = 0.01$ and $p_u' = 0.10$, respectively. From Table 4, the area under the Normal curve below -2.33 standard deviations from the mean equals 0.01, and the area below -1.28 standard deviation equals 0.10. Thus, an <u>acceptable</u> soil having $p_a' = 0.01$ and standard deviation $s_x' = 2\%$ would have a mean,

$$m_a = L + 2.33\, s_x'$$
$$= 95\% + 2.33(2\%) \qquad\qquad -63-$$
$$= 99.7\%;$$

and an unacceptable soil having $p_u' = 0.10$ would have a mean,

$$m_u = L + 0.84 s_x'$$

$$= 95\% + 1.28(2\%)$$ -64-

$$= 97.6\%.$$

The test index z is calculated from Eqn. 56. Due to random sampling variability, the value of z varies from one sample of n tests to another even for the same soil. This sampling variability can be characterized by a mean m_z and standard deviation s_z for each of the soils above. Specifically, for the acceptable quality soil,

$$m_z = \frac{m_a - L}{s_x} = \frac{99.7-95}{2} = 2.35$$ -65-

$$s_z = 1/\sqrt{n} .$$ -66-

For the unacceptable quality soil, $m_z = 1.28$ and $s_z = 1/\sqrt{n}$. These are the means and standard deviations that the test statistic z would have if the actual soil being tested were just at the edge of being acceptable or just at the edge of being unacceptable, respectively.

The Buyer's risk and Seller's risk specify target probabilities of accepting the two types of soil above when using the acceptance sampling plan. For acceptable soil $Pr\{z<z^*\} = \alpha$; for unacceptable soil $Pr\{z>z^*\}=\beta$. This gives two equations. Again from Table 4, for $Pr\{z<z^*\} = \alpha = 0.05$ the mean of z must be 1.645 standard deviation larger than z*,

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$$m_{za} - 1.645 \, s_{za} = z*$$

$$2.33 - 1.645/\sqrt{n} = z* \quad .$$

-67-

For $\Pr\{z > z*\} = \beta = 0.10$, the mean of z must be 1.28 standard deviations smaller than $z*$,

$$m_{zu} + 1.28 \, s_{zu} = z*$$

$$1.28 + 1.28/\sqrt{n} = z* \quad .$$

-68-

Eqns. 67 and 68 are solved simultaneously to give,

$$n = 7.79 \rightarrow \text{say, } 8$$

-69-

$$z* = 1.75$$

-70-

An example of the acceptance sampling plan is specified as shown in Plate 7.

The design of an acceptance sampling scheme may be accomplished more quickly by algebraically solving for the sample size n and acceptance criterion $z*$. Define,

$z_{1-\alpha}$ = standardized Normal variable for which the probability of not being exceeded (Table 6) is $1-\alpha$.

$z_{1-\beta}$ = standardized Normal variable for which the probability of not being exceeded (Table 6) is $1-\beta$.

z_a = standardized Normal variable for which the probability of not being exceeded (Table 6) is $1-p_a'$

z_u = standardized Normal variable for which the probability of not being exceeded (Table 6) is $1-p_u'$

For a single criterion acceptance sampling plan having parameters (p_a', α) and (p_u', β) the sample size and acceptance criterion are,

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$$n = \left(\frac{z_{1-\alpha} + z_{1-\beta}}{z_a - z_u}\right)^2 \qquad\qquad -71-$$

$$z^* = \frac{z_a\,z_{1-\beta} + z_u\,z_{1-\alpha}}{z_{1-\alpha} + z_{1-\beta}} \qquad\qquad -72-$$

To summarize, the procedure for designing a single limit acceptance sampling plan is:

1. Select a Seller's risk α, and a Buyer's risk β.
2. Select acceptable quality level p_a' and unacceptable quality level p_u'.

3. Find values for standard Normal variables corresponding to $1-\alpha$, $1-\beta$, $(1-p_a')$, and $(1-p_u')$ probabilities of not being exceeded ($z_{1-\alpha}$, $z_{1-\beta}$, z_a, z_u).

4. Calculate the sample size by

$$n = \left(\frac{z_{1-\alpha} + z_{1-\beta}}{z_a - z_u}\right)^2 \qquad\qquad -73-$$

5. Calculate the acceptance criterion by

$$z^* = \frac{z_a\,z_{1-\beta} + z_u\,z_{1-\alpha}}{z_{1-\alpha} + z_{1-\beta}} \qquad\qquad -74-$$

6. Plot the OC curve

Single Limit with Unknown Standard Deviation

Usually the standard deviation of the material property being tested is unknown. The only information about the standard deviation comes from the data themselves in the form of the sample standard deviation,

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - m_x)^2} \qquad\qquad -75-$$

If the sample size is large (n>20), the sample standard deviation will be close to the real standard deviation and the assumption of known standard deviation can be made with neglible error. If the sample size is not large, a slight modification to the foregoing procedure must be made.

When the standard deviation is unknown the quantity z is calculated using the sample standard deviation s_x,

$$z = \frac{m_x - L}{s_x} . \qquad\qquad -76-$$

Whereas, when the standard deviation is known the quantity z has a Normal distribution, when the sample standard deviation is substituted for the real standard deviation the calculated value of z has more variability. Now the denominator as well as the numerator will vary from one sample to another. The frequency distribution of z takes on the slightly broader shape of the Student -t distribution.

The procedure for designing sampling plans and calculating OC curves when the standard deviation is unknown is the same as when the standard deviation is known, with the exception that tables of the Student -t frequency distribution rather than the Normal distribution are used. Areas under the Student -t distribution for the standardized case of zero-mean and unit standard deviation are given in Table 6. Note, unlike the Normal distribution, Student -t depends on the sample size n. As n becomes large the shape of the Student -t approaches the Normal distribution.

Convenient approximations for sample size and acceptance criterion when the standard deviation is unknown are (Wallis, 1947),

$$ z^* = \frac{z_a \, z_{1-\beta} + z_u \, z_{1-\alpha}}{z_{1-\alpha} + z_{1-\beta}} \qquad -77- $$

$$ n = \left(1 + z^{*2}/2\right) \left(\frac{z_{1-\alpha} + z_{1-\beta}}{z_a - z_u}\right)^2 \qquad -78- $$

Thus, when the standard deviation is not known a larger sample must be taken to get the same OC curve. The sample size must be larger by the factor $(1+z^{*2}/2)$.

The example of the previous section is recalculated in Plate 8, now relaxing the assumption that the standard deviation is known. The OC curve can be calculated approximately but acceptably by assuming z to be Normally distributed with a standard deviation equal to $s_x \left(1/n + (z^{*2}/2n)\right)^{1/2}$. The

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approximate OC curve is shown in Fig. 39. Thus, to calculate the real fraction defective p' corresponding to a given probability of acceptance q (i.e, to plot the OC curve), first the corresponding standardized Normal deviation z_q is taken from Table 4. Next, z_q is increased by the factor $\left(1/n + (z^{*2}/2n)\right)^{1/2}$. Then a corresponding z_p' is calculated as

$$z_p' = z^* - z_q \left(1/n + (z^{*2}/2n)\right)^{1/2} \qquad . \qquad \text{-79-}$$

Then Table 4 is used to determine p'. For example, in Plate 7, $z^* = 1.63$ and $n = 6$.

Double Specification Limits

The preceding plan pertains to the case of one specification limit. For example, dry density should be at least 95% standard or modified Procter maximum. When deviations in either direction are important the plan must be modified. For a lower limit of acceptability L and an upper limit U, the minimum fraction defective occurs when m_x lies halfway between L and U; that is, when the limits are symmetric about the mean. In this case,

$$z_L = \frac{m_x' - L}{s_x'}, \qquad \text{-80-}$$

$$z_U = \frac{U - m_x'}{s_x'}, \text{ and} \qquad \text{-81-}$$

$$z = \frac{z_L - z_U}{2} = \frac{U-L}{2s_x'} \; . \qquad \qquad -82-$$

The fraction defective equals the area under the Normal curve outside $\pm z$, or twice the fraction defective read from Table 4. Note that z depends only on the upper and lower limits U, L and on s_x'. It does not depend on m_x'. Thus, if s_x' is known, the first step is to assume that the acceptable fraction defective is greater than the area under the Normal curve outside $\pm z$. This may be done without sampling, and indicates whether the variability of the construction process reflected in s_x' is so large as to preclude any possibility of the tested soil being found acceptable.

Presuming that z from Eqn. 80 is sufficiently large that rejection is not inescapable, the fraction defective will depend on both z_L and z_U, and the acceptance criterion must be based on both. In concept, this is done by summing the fraction defective beneath L and the fraction defective above U and comparing that sum to the criterion M. However, a simpler procedure can be developed by considering the operating characteristic curve of the sampling plan.

Double Limits, Standard Deviation Known

The fraction defective for double specification limits is that proportion of the area under the frequency distribution of the material property which lies either below a lower specification limit,

L = lower specification limit,

or above an upper specification limit,

$$U = \text{upper specification limit.}$$

For constant standard deviation the fraction defective is minimized when the mean m_x' lies halfway between L and U. In this case,

$$-(L-m_x')/s_x' = (U-m_x')s_x' = (U-L)/2s_x' \qquad -83-$$

Thus, a quick check should be made to see whether a material can possibly meet the fraction defective double specification standard by finding the area under the Normal frequency distribution outside $\pm z = (U-L)/2s_x'$. If this area is greater than the acceptable fraction defective p_a' no sampling plan alone will assure quality. The construction process must be changed to make the material more uniform and thus reduce s_x'.

In the general case for double specification limits, an acceptance sampling plan to assure fraction defective follows the following procedure:

    1.   Take a sample of size n

    2.   From the results, calculate the sample mean
        $m_x = (1/u)\Sigma x_i$

    3.   Compute the quantities

$$z_L = \frac{m_x - L}{s_x'} \qquad -84-$$

$$z_U = \frac{U - m_x}{s_x'} \qquad -85-$$

4. Specify an acceptance criterion z^*: If $z_L \geqslant z^*$ and $z_U \geqslant z^*$, then accept otherwise, reject.

The problem with double specification limits is determining z^*. In the case of single specification limit z^* was determined from areas under the Normal frequency distribution to one side of a specification limit. In the double specification case z^* must be determined from the sum of the areas above U and below L.

Consider the problem of acceptance sampling for compaction water content. The target value is Proctor optimum water content. The upper specification limit is U = +2% optimum; the lower specification limit is L = -2%. Presuming the standard deviation of water content to be 1%, the limits are

$$\pm z = \pm \frac{U-L}{2s_x} = \pm \frac{+2 - (-2)}{2(1)} = \pm 2 \qquad\qquad -86-$$

That is L and U are 4 (i.e., ±2) standard deviations apart. From Table 4 the area under the Normal curve beyond z=2 is 0.02. Thus, the lowest possible fraction defective would be twice 0.02 or about 4%. The fraction defective for values of the mean other than that halfway between U and L are shown in Table 8.

Presume for sake of example that the acceptable quality level or AQL expressed as a fraction defective were $p_a' = 0.10$. That is, the lift would be considered acceptable if at least 90% of the soil had a compaction water content between L = -2% Proctor optimum and U = +2%. From Table 8 (by interpolating m_x' values) any lift with an average water content bet een -0.7%

and +0.7% would be acceptable, for the fraction of any of these lifts with water contents outside ±2% would be less than 0.10. At $m_x' = -0.7\%$ the fraction below -2% is 0.096 and the fraction above +2% is 0.004. The sum is 0.10. Similarly but in reverse at $m_x' = +0.7\%$, the fraction above +2% is 0.096 and the fraction below -2% is 0.004. The double limit specification can thus be met by combining two single limit tests designed such that the acceptable fraction defective in each is reduced from $p_a' = 0.10$ to $p_a' = 0.096$. One applies on the upper limit side, the other applies on the lower limit side. The design for these two plans is exactly as discussed before, and is carried out in Plate 9.

Double Limits, Standard Deviation Unknown

The problem of designing an acceptance sampling plan for fraction defective with double specification limits and unknown standard deviation is less easily solved than the single limit problem. In particular, with double limits the shape of the OC curve depends on how the fraction defective is split between the upper and lower tail of the distribution. However, the availability of statistical tables and graphs designed expressly for the purpose (US DOD Military Standard 414) greatly simplifies the task. For the purpose of acceptance sampling of engineered fills, the graphs of Fig. 41 and Fig. 42 provide sufficient accuracy.

The procedure begins as for the single limit, unknown standard deviation case. Buyer's and Seller's risk α and β are specified, and acceptable and unacceptable fractions defective p_a' and p_u'. Eqns. 77 and 78 are used to

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estimate a sample size n and an acceptance criterion z*. From the estimates of n and z* the quantity

$$Y = \frac{1 - \frac{z^* \sqrt{n}}{n - 1}}{2}$$ -87-

is calculated and used to enter the absicca of Fig. 41. On the ordinate, and corresponding to the appropriate value of n, an allowable fraction defective M is read.

The test procedure is implemented by taking a sample size n, calculating the sample mean m_x and sample standard deviation s_x, and then computing the test indices

$$z_L = \frac{m_x - L}{s_x}$$ -88-

$$z_U = \frac{U - m_x}{s_x}$$ -89-

From Fig. 42, for the appropriate value of n, estimated fractions defective corresponding to z_L and z_U are read of as p_L and p_U, respectively. These are summed to obtain an estimate of the total fraction defective, which is in turn compared to M to decide whether to accept or reject the lift. If $p_L + p_U < M$, then the lift is accepted; otherwise, the lift is rejected. An example is given in Plate 10. The OC curve for this procedure is approximately the same as in the single limit case, using the same values of α, β, p_a', and p_u'.

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Table 8 -- Fraction defective for double sampling limits.

m_x'	$\dfrac{L-m_x}{s_x}$	$\dfrac{U-m_x}{s_x}$	P_L	P_U	$P=PL+PU$
+2.0	4.0	0.0	–	0.50	0.50
+1.5	3.5	0.5	–	0.31	0.31
+1.0	3.0	1.0	0.001	0.16	0.17
+0.5	2.5	1.5	0.006	0.07	0.086
0.0	2.0	2.0	0.02	0.02	0.04
−0.5	1.5	2.5	0.07	0.006	0.086
−1.0	1.0	3.0	0.16	0.001	0.17
−1.5	0.5	3.5	0.31	–	0.31
−2.0	0	4	0.50	–	0.50

Note: range U-L is kept constant.

PLATE 3

SUBJECT: Acceptance sampling plan to assure mean value of
compacted dry density; standard deviation known.

I. PROBLEM:

Design an acceptance sampling plan to assure the value of mean
compacted dry density of an engineered fill.

II. SOLUTION:

1. Parameters:

Seller's risk	α	= 0.05
Buyer's risk	β	= 0.10
AQL	m_a	= 120 pcf
UQL	m_u	= 110 pcf
Std. Dev.	$s_x{'}$	= 10 pcf

2. Find standard Normal variable corresponding to $1-\alpha$ and β:

$$z_{1-\alpha} = z_{0.95} = +1.65$$
$$z_\beta = z_{0.10} = -1.28$$

3. Set Seller's risk and Buyer's risk:

$$\frac{m^* - m_a}{s_x{'}/\sqrt{n}} = \frac{m-120 \text{ pcf}}{10 \text{ pcf}/\sqrt{n}} = {}^{-z}1-\alpha = -1.65$$

$$\frac{m^* - m_u}{s_x{'}/\sqrt{n}} = \frac{m-110 \text{ pcf}}{10 \text{ pcf}/\sqrt{n}} = {}^{-z}\beta = 1.28 .$$

4. Solve simultaneously to obtain:

$$n = 8.6 \rightarrow 9$$
$$m^* = 114 \rightarrow 114 \text{ pcf.}$$

5. OC curve shown as Fig. 29.

PLATE 4   SHEET 1/2

SUBJECT: Acceptance sampling plan to assure mean value of compacted dry
           density; standard deviation unknown.

I.   PROBLEM: Design acceptance sampling plan to assure mean value of compacted
     dry density.

II. SOLUTION:

     1.   Specify,

               α = 0.05
               β = 0.10
               m_a = AQL = 120 pcf
               m_u = UQL = 110 pcf

     2.   Estimate Standard Deviation

               $s_x' \approx 10$ pcf.

     3.   Estimate Sample Size

          $$\lambda = \frac{m_a - m_u}{s_x'} = \frac{120 - 110 \text{ pcf.}}{10 \text{ pcf}} = 1.0$$

          For $\lambda=1.0$ the probability of accepting should be $\beta=0.10$.   Fig. 32
          shows that n=10 is the approximate sample size providing this
          probability of accepting.

     4.   Find t*,

          From Table 6 the value of t which is exceeded $(1-\alpha)= 0.95$ fraction
          of the time is t*=-1.83.

PLATE 4   SHEET 2/2

SUBJECT: Acceptance sampling plan to assure mean value of compacted dry
           density; standard deviation unknown.

5.   Specify acceptance sampling,

   a)   Take a sample of size n=10
   b)   Calculate mean of sample $m_x = (1/n) \sum x_i$
        Calculate standard deviation of
             sample $s_x = (1/n-1) \sum (x_i - m_x)^2$
   c)   compute quantity

$$t = \frac{m_x - m_a}{s_x/\sqrt{n}}$$

$$= \frac{m_x - 120 \text{ pcf}}{s_x/\sqrt{10}}$$

   d)   if $t \geq -1.83$, then accept
        $t < -1.83$, then reject

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PLATE 5   SHEET 1/2

SUBJECT: Design acceptance sampling plan to assure average compaction water
content within double specification limits; standard deviation is
known.

I.   PROBLEM:

Design acceptance sampling plan to assure average compaction
water content is within ±2% Procter optimum.   Standard deviation is known
to be $s_x' = 2\%$.

II.   SOLUTION:

1.   Specify:

$\alpha = 0.05$ Target water content = Procter optimum.
$\beta = 0.05$ UQL_U = +2% Procter optimum.
$s_x' = 2\%$ UQL_L = -2% Procter optimum.

2.   Write equations for Seller's risk and Buyer's risk:

$$\frac{m_U^* - m_a}{s_x'/\sqrt{n}} = z_{1-\alpha/2} \quad \rightarrow \quad \frac{m_U^* - o}{2/\sqrt{n}} = 1.96$$

$$\frac{m_L^* - m_a}{s_x'/\sqrt{n}} = z_{\alpha/2} \quad \rightarrow \quad \frac{m_L^* - o}{2/\sqrt{n}} = -1.96$$

$$\frac{m_U^* - UQL_U}{s_x'/\sqrt{n}} = z_\beta \quad \rightarrow \quad \frac{m_U^* - 2\%}{2/\sqrt{n}} = -1.645$$

$$\frac{m_L^* - UQL_L}{s_x'/\sqrt{n}} = z_{1-\beta} \quad \rightarrow \quad \frac{m_U^* - (-2\%)}{2/\sqrt{n}} = 1.645$$

PLATE 5   SHEET 1/2

SUBJECT: Design acceptance sampling plan to assure average compaction water
           content within double specification limits; standard deviation is
           known.

3.   Solve equations simultaneously:

a) from Eqns. 1 and 2,

$$m_U^* = -m_L^*$$

b) from Eqns. 1 and 3,

$$n \approx 13$$

c) from Eqns. 1,2,3,4,

$$m_U^* = -m_L^* = 1.09$$

4.   Specify sampling plan:

a) Take sample of size n=13 water contents
b) Calculate mean of sample $m_x = (1/n)\Sigma x_i$
c) If   -1.09% < m_x < +1.09% Procter optimum, then accept lift.
d) If m_x > 1.09% or m_x < 1.09% Procter optimum, then reject
         lift.

NOTE: The relatively large sample size and tight acceptance limits for this
inspection plan are caused by the large variability of the soil relative to the
unacceptable quality limits of ±2% Proctor optimum water content.

PLATE 6

SUBJECT: Operating characteristic (OC) curve for fraction defective sampling, typical calculation; standard deviation known.

1. PROBLEM: For an acceptance sampling plan specified by

$$n = 5 \; ,$$
$$L = 95\% \text{ optimum Proctor,}$$
$$s_x' = 2\%,$$
$$z^* = 1.645;$$

Find the probability of accepting material actually having 5% defective (i.e., 5% of the material compacted to less than L).

2. SOLUTION:

1 Find actual mean if $s_x' = 2\%$ and fraction defective is $P' = 5\%$.

From Table 4, 5% defective corresponds to a mean 1.645 standard deviations greater than the lower limit of acceptable material.

$$m_x' = L + 1.645 \; s_x'$$
$$= 95\% + 1.645 \; (2\%) = 98.3\%$$

2 Find mean and standard deviation of z for n=5, $m_x' = 98.3$, L = 95%, and $s_x' = 2\%$

$$m_z = (m_x' - L)/s_x'$$
$$= (98.3\% - 95\%)/2\% = 1.645$$

$$s_z = 1/\sqrt{n}$$
$$= 1/\sqrt{5} = 0.45$$

3 Find number of standard deviations of z separating m_z from $z^* = 1.645$

$$\text{number standard deviations} = (m_z - z^*)/s_z$$
$$= (1.65 - 1.645)/0.45$$
$$\cong 0.00$$

4 Find probability of accepting material if m_z is 0 standard deviations above z^*.

From Table 4, $z = 0.00 \rightarrow \text{Pr}\{\text{rejecting}\} = 0.50$
$\text{Pr}\{\text{accepting}\} = 1 - \text{Pr}\{\text{rejecting}\}$
$$= 1 - 0.50$$
$$= 0.50.$$

PLATE 7   SHEET 1/3

SUBJECT: Acceptance sampling plan for fraction defective; standard deviation
           known.

1.   PROBLEM:

Design an acceptance sampling plan for fraction defective
%-compaction, when the standard deviation is known.

2.   SOLUTION:

1.   Specify: Buyer's Risk α = 0.05
                Seller's Risk β = 0.10
                Acceptable fraction defective p_a' = 0.01
                Unacceptable fraction defective p_u' = 0.10
                Standard deviation s_x' = 2% Procter optimum
                Lower Limit of acceptable compaction L=95% Procter
                optimum

2.   Calculate average percent compaction corresponding to p_a' and p_u':

Acceptable compaction: $\dfrac{m_a - 95\%}{2\%}$ = $z_{0.99}$ = 2.33 $\rightarrow$ m_a = 99.7%

Unacceptable compaction; $\dfrac{m_u - 95\%}{2\%}$ = $z_{0.90}$ = 1.28 $\rightarrow$ m_u = 97.6%

3.   Define test index:

$$z = \frac{m_x - L}{s_x'}$$

PLATE 7   SHEET 2/3

SUBJECT: Acceptance sampling plan for fraction defective; standard deviation
           known.

4.   Calculate mean and standard deviation of z for acceptable and
     unacceptable compaction:

   Acceptable compaction $m_{za} = \dfrac{m_a - 95\%}{2\%} = 2.33$

                                $s_{za} = 1/\sqrt{n}$

   Unacceptable compaction $m_{zu} = \dfrac{m_u - 95\%}{2\%} = 1.28$

                                $s_{zu} = 1/\sqrt{n}$

5.   Fix Buyer's and Seller's risk:

   Seller's risk (acceptable compaction): $\Pr\{z<z^*\} = \alpha; = 5\%$

   $$\frac{z^* - m_{za}}{s_z} = z_{0.05} = -1.645$$

   $$\frac{z^* - 2.33}{1/\sqrt{n}} = -1.645$$

   Buyer's risk (unacceptable compaction): $\Pr\{z>z^*\} = \beta = 0.10$

   $$\frac{z^* - m_{zu}}{s_{zu}} = z_{1-0.10} = +1.28$$

   $$\frac{z^* - 0.84}{1/\sqrt{n}} = +1.28$$

PLATE 7   SHEET 3/3

SUBJECT: Acceptance sampling plan for fraction defective; standard deviation
           known.

Solve simultaneously to obtain:

$n = 7.8 \rightarrow 8$

$z^* = 1.74$

6.   Specify acceptance sampling plan:

   a.   Perform $n = 8$ density tests
   b.   Calculate the mean of the four results, m_x.
   c.   Calculate the quantity z,

$$z = \frac{m_x - L}{s_x'} = \frac{m_x - 95\%}{2\%}$$

   d.   If,

       $z \geqslant 1.74$, then accept
       $z < 1.74$, then reject

The OC curve for the sampling plan ($n=8$, $z^*=1.74$) is shown in Fig.
38.

PLATE 8   SHEET 1/2

SUBJECT: Acceptance sampling plan for fraction defective; standard deviation
           unknown.

PROBLEM: Design an acceptance sampling plan for soil compaction with the
           properties

α = 0.05 ρ_a' = 0.01
β = 0.10 ρ_u' = 0.20
L   = 95% optimum Procter
s_x = Unknown.

SOLUTION:

1.   <u>Find standard Normal variables corresponding to $\alpha, \beta, \rho_a', \rho_u'$.</u>

From Table 4: $\alpha \rightarrow z_{1-\alpha} = 1.645$
$\beta \rightarrow z_{1-\beta} = 1.28$
$p_a' \rightarrow z_a = 2.33$
$p_u' \rightarrow z_u = 0.84$

2.   <u>Calculate sample size n and acceptance criterion Z^*.</u>

$$z^* = \frac{z_a \, z_{1-\beta} + z_u \, z_{1-\alpha}}{z_{1-\alpha} + z_{1-\beta}}$$

$$= \frac{(2.33)\,(1.28) + (0.84)\,(1.645)}{1.28 + 1.645}$$

$$= 1.5$$

$$n = (1 + \frac{z^{*2}}{2}) \; (\frac{z_{1-\alpha} + z_{1-\beta}}{z_a - z_u})^2$$

$$= (1 + \frac{1.49^2}{2}) \; (\frac{1.645 + 1.28}{2.33 - 0.84})$$

$$= 8.13 \rightarrow 8$$

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PLATE 8   SHEET 2/2

SUBJECT: Acceptance sampling plan for fraction defective; standard deviation
           unknown.

3.   Specify Sampling Plan

   a.   Take random sample of size n = 8
   b.   Compute sample mean m_x, standard deviation s_x, and the test
        index

$$z_L = \frac{m_x - L}{s_x}$$

   c.   Compare z_L with $z^* = 1.5$,
           If $z_L > 1.5$, then accept lift.
           If $z_L < 1.5$, then reject lift.

4.   The OC curve for this plan is shown in Fig. 39.

PLATE 9   SHEET 1/2

SUBJECT: Acceptance sampling plan to assure fraction defective with double
           specification limits; standard deviation known.

I. PROBLEM:

Design an acceptance sampling plan to assure fraction defective on
the basis of compacted water content. Assume SD known to be 1% optimum
Proctor.

II. SOLUTION:

1. Specify,

$$\alpha = 0.05 \qquad s_x' = 1\%$$
$$\beta = 0.10 \qquad U = +2\% \text{ Procter optimum}$$
$$p_a' = 0.10 \qquad L = -2\% \text{ Procter optimum}$$
$$p_u' = 0.30$$

2. Determine values of m_x' such that the sum of fraction defective above
    U and fraction defective below L equals the AQL, $P_a'=0.10$

from Table 8 (by interpolation):

mean	p_U	p_L	P TOTAL
$m_x' = +0.7$	0.096	0.004	0.10
$m_x' = -0.7$	0.004	0.096	0.10

3. Determine standard Normal variables corresponding to $(1-\alpha)$, β,
    maximum of (p_U, p_L), and p_a'

$$\alpha = 0.05 \qquad z_{1-\alpha} = 1.65$$
$$\beta = 0.10 \qquad z_{1-\beta} = 1.28$$
$$\max(p_U', p_L') = 0.096 \qquad z_{pa} = 1.30$$
$$p_u' = 0.30 \qquad z_{pu} = 0.53$$

PLATE 9   SHEET 2/2

SUBJECT: Acceptance sampling plan to assure fraction defective with double
           specification limits; standard deviation known.

---

4.   Evaluate n and z*,

From Eqn. 71 $n = \left(\dfrac{z_{1-\alpha} + z_{1-\beta}}{z_a - z_u}\right)^2$

$= \left(\dfrac{+1.65+1.28}{1.30-0.53}\right)^2 \quad \to 14.5 \to 15$

From Eqn. 72 $z^* = \dfrac{z_a\, z_{1-\beta} + z_u\, z_{1-\alpha}}{z_{1-\alpha} + z_{1-\beta}}$

$= \dfrac{(1.30)(1.28)+(0.53)(1.65)}{(1.65)+(1.28)} \quad = 0.87$

5.   Specify acceptance sampling plan
     a)   Take random sample of size 15
     b)   Calculate sample mean m_x, and test indices,

$z_U = \dfrac{U - m_x}{s_x'}$

$z_L = \dfrac{m_x - L}{s_x'}$

     c)   Compare z_u and z_L with $z^* = 0.87$,
          If z_u and $z_L > z^*$, then accept
          If z_u and $z_L < z^*$, then reject

6.   OC Curve is shown in Fig. 40.

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PLATE 10   SHEET 1/2

SUBJECT: Acceptance sampling plan to assure fraction defective with double
           specification limits; standard deviation unknown

I. PROBLEM:

Design an acceptance sampling plan to assure fraction defective on
the basis of compaction water content.   Standard deviation is unknown

II. SOLUTION:

1. Specify,

α = 0.10   U = +2% Procter optimum
β = 0.10   L = -2% Procter optimum
p_a' = 0.05
p_u' = 0.30

2. Determine standard Normal deviates corresponding to α, β, p_a' and p_u'

α = 0.10 $z_{1-\alpha}$ = +1.28
β = 0.10 $z_{1-\beta}$ = +1.28
p_a' = 0.05 z_a = 1.65
p_u' = 0.30 z_u = +0.53

3. Use Eqns. 77 and 78 to estimate n and z*,

$$n = (1+z*^2/2) \left(\frac{z_{1-\alpha} + z_{1-\beta}}{z_a - z_u}\right)^2 = (1+ \frac{1.02^2}{2}) \left(\frac{+1.28+1.28}{1.65-0.53}\right)^2 = 7.94 \to 8$$

$$z* = \frac{z_a z_{1-\beta} + z_u z_{1-\alpha}}{z_{1-\alpha} + z_{1-\beta}} = \frac{(1.65)(1.28)+(0.53)(1.28)}{(1.28)+(1.28)} = 1.09$$

4. Estimate allowable fraction defective M

Compute quantity

$$y = \frac{1 - \dfrac{z* \sqrt{n}}{(n-1)}}{2} = \frac{1-(1.09)\sqrt{8}|(8-1)}{2} = 0.28$$

Read M from Fig. 41 for y=0.28, n=8   →   M = 0.15

PLATE 10   SHEET 2/2

SUBJECT: Acceptance sampling plan to assure fraction defective with double
          specification limits; standard deviation unknown

5. Specify test procedure

    a. Take a random sample of size n=8
    b. Compute sample mean m_x, and standard deviation s_x, calculate
        test indices,

$$z_U = \frac{U - m_x}{s_x}$$

$$z_L = \frac{m_x - L}{s_x}$$

    c. From Fig. 42 read values of p_a and p_u corresponding to z_U and
        z_L

    d. Compare $p_L + p_U$ to M,

        If $P_L + p_U \leqslant M$, then accept.
        If $P_L + p_U < M$, then reject.

6. The OC curve for this plan is shown in Fig. 43
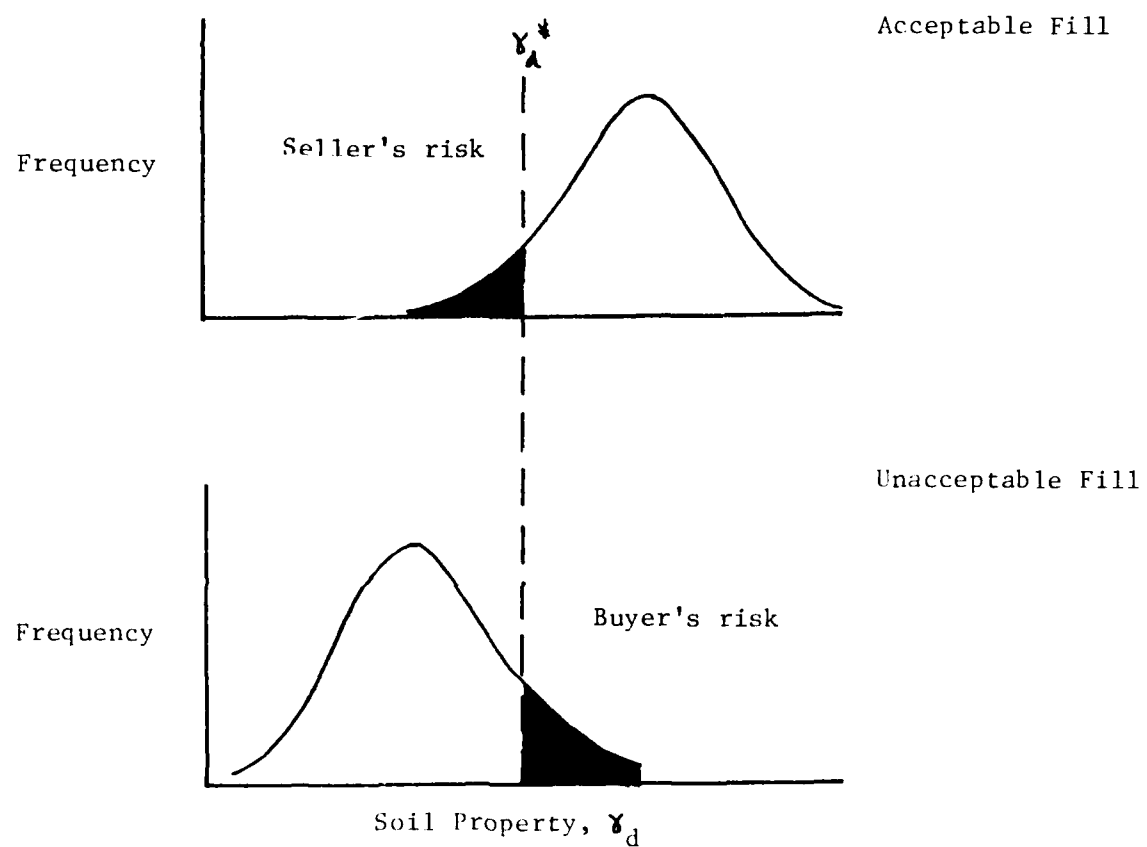
Figure 28 -- Frequency distribution of test results taken from acceptable and unacceptable fills showing Seller's and Buyer's risks (schematic).
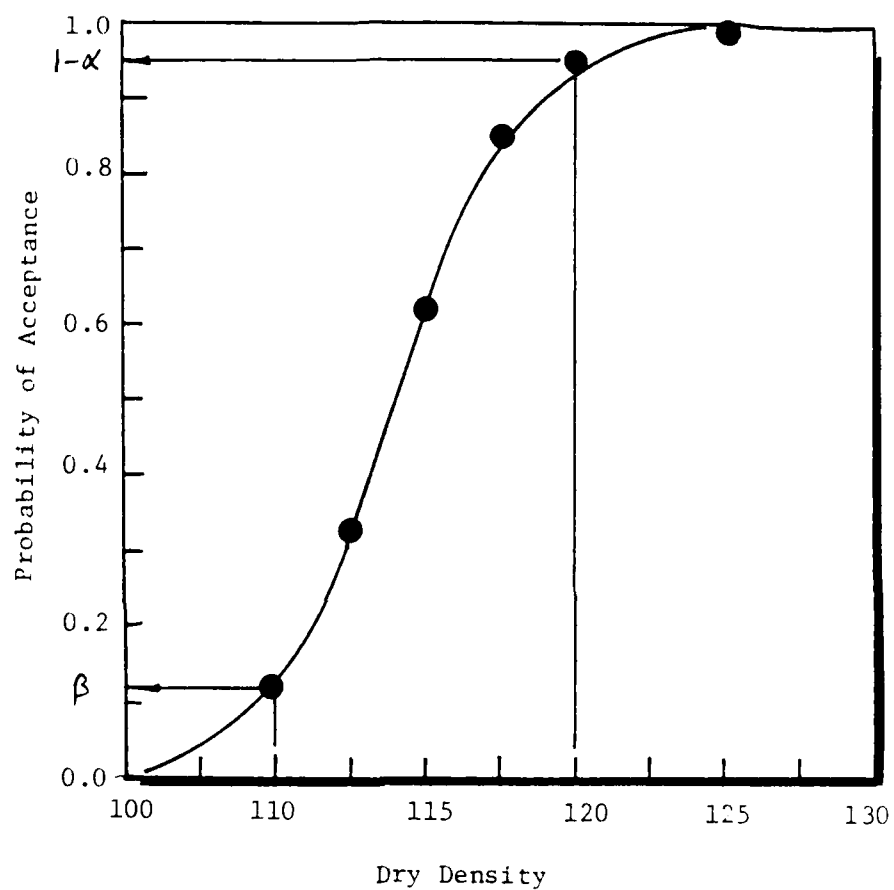
Figure 29 -- Operating characteristic (OC) curve for an acceptance
sampling plan to assure the value of mean compacted dry density of
an engineered fill (see Plate 3).
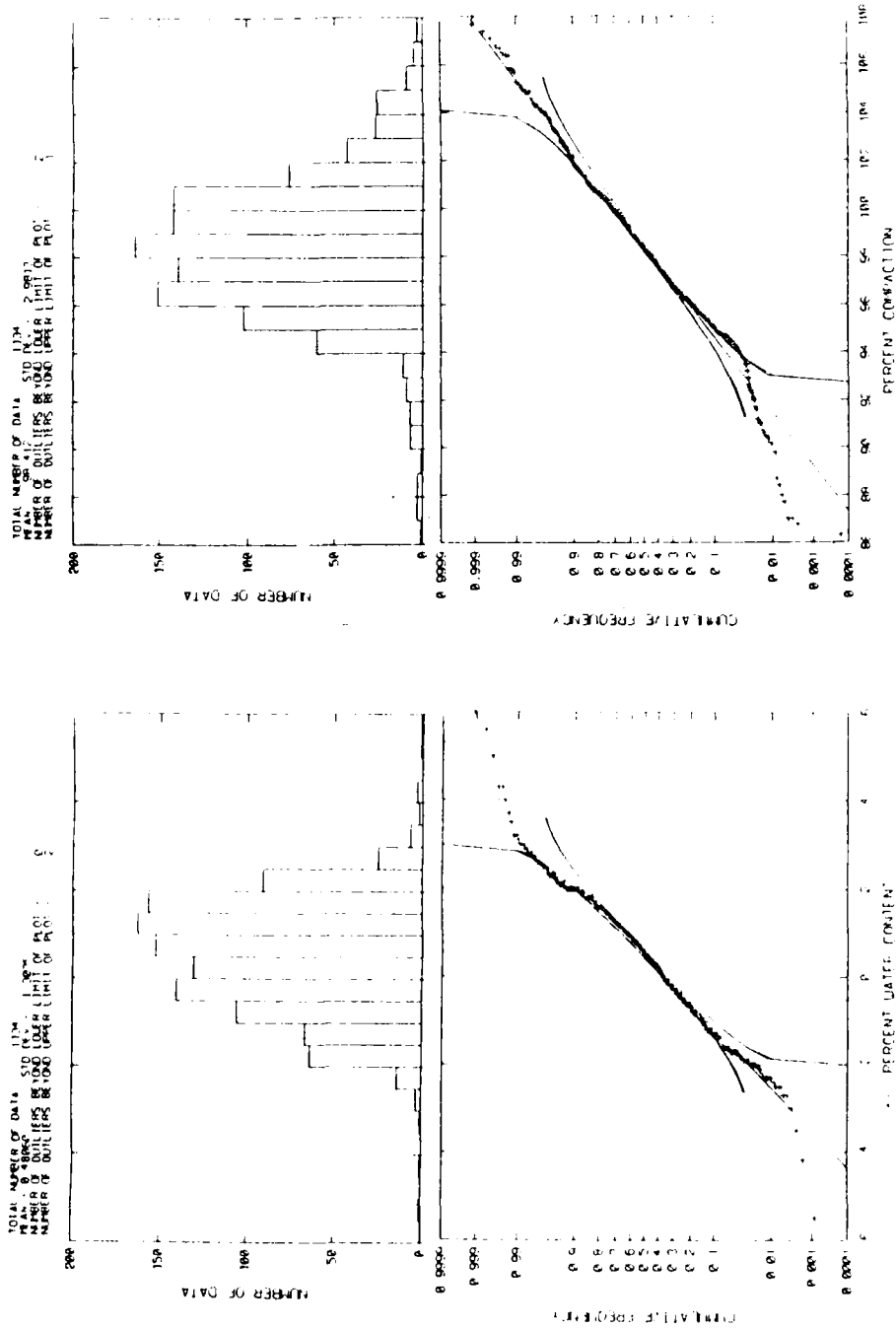
Figure 30 -- Examples of empirical compaction control data showing closeness to
Normal distributions of frequency.

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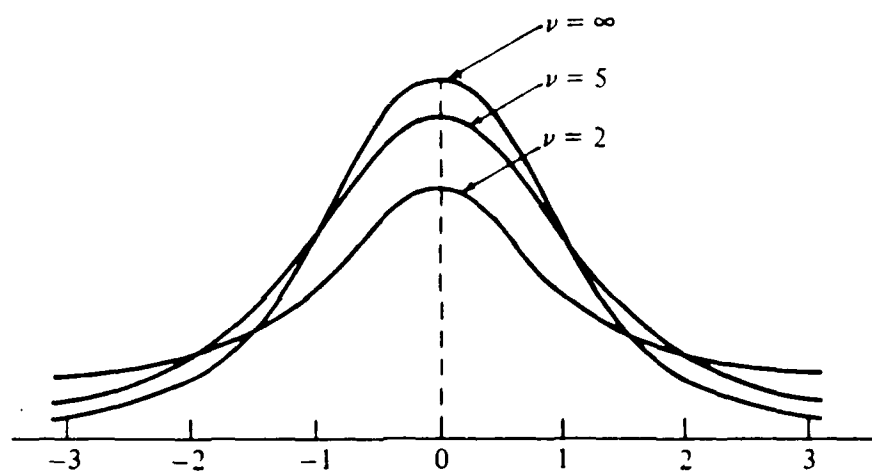
The t distribution curves for $v = 2, 5$, and ∞.

Figure 31 -- Student's-t distribution.

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FIGURE 15.9

Operating Characteristic Curves for Single-Limit Sampling Plans Based on the Statistic

$$t = \frac{\bar{X} - \bar{X}_0'}{s/\sqrt{n}} \text{ with } \alpha = 0.05*$$

$$\lambda = \frac{|\bar{X}_1' - \bar{X}'|}{\sigma'} \text{ OR } \frac{|\bar{X}_0' - \bar{X}'|}{\sigma'}$$
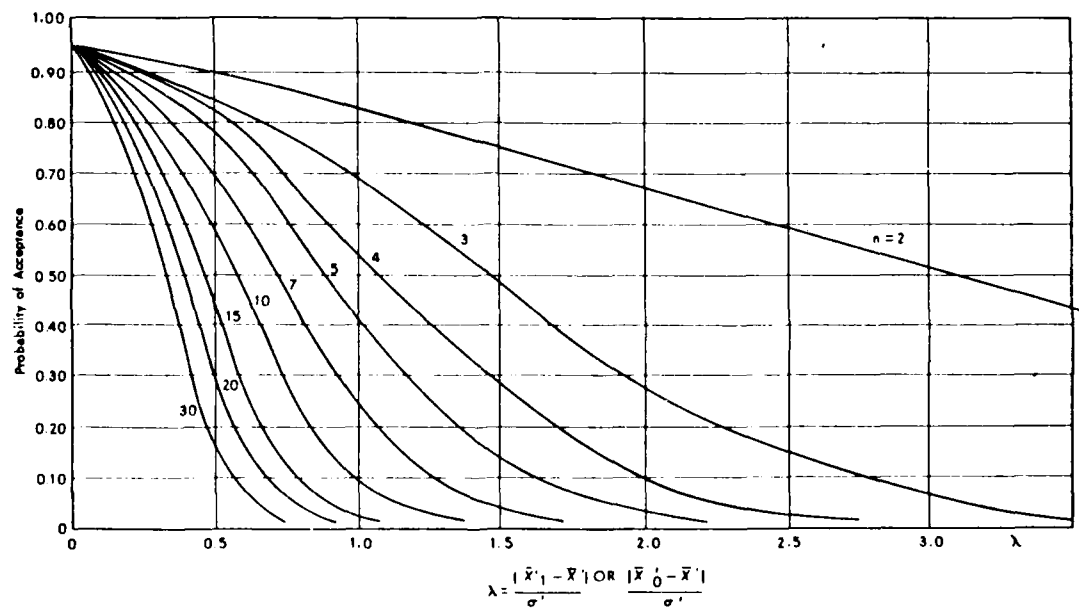
* In acceptance sampling. $\bar{X}_0'$ the $\bar{X}_0'$ of the plan and $\bar{X}'$ any other lot or process quality. n size of sample. The lot or process is assumed to be normally distributed or approximately normally distributed. The sampling plan has only one acceptance limit. Source of original data: J. Neyman and B. Tobarska. "Errors of the Second Kind in Testing 'Student's' Hypothesis." *Journal of the American Statistical Association*, Vol. XXXI, pp. 318-26.

Figure 32 -- Operating characteristic curves corresponding to various sample sizes. From Duncan (1974).

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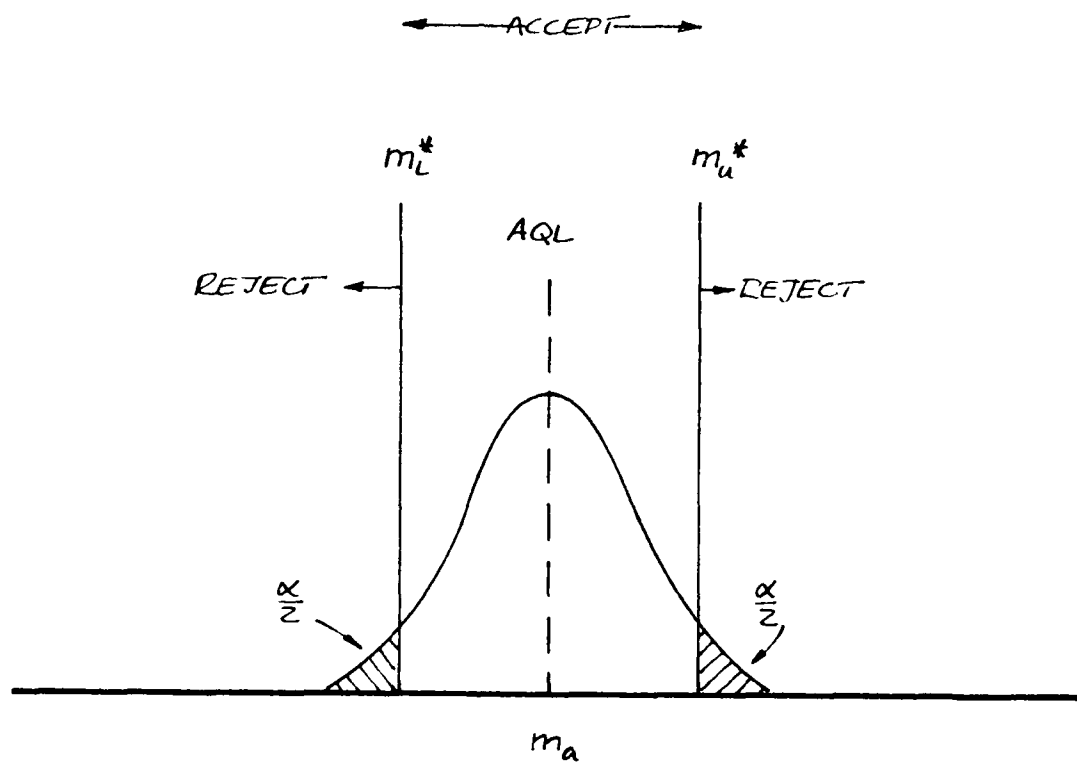
Figure 33 -- Frequency distribution of the sample average m_x for a sample of size n.
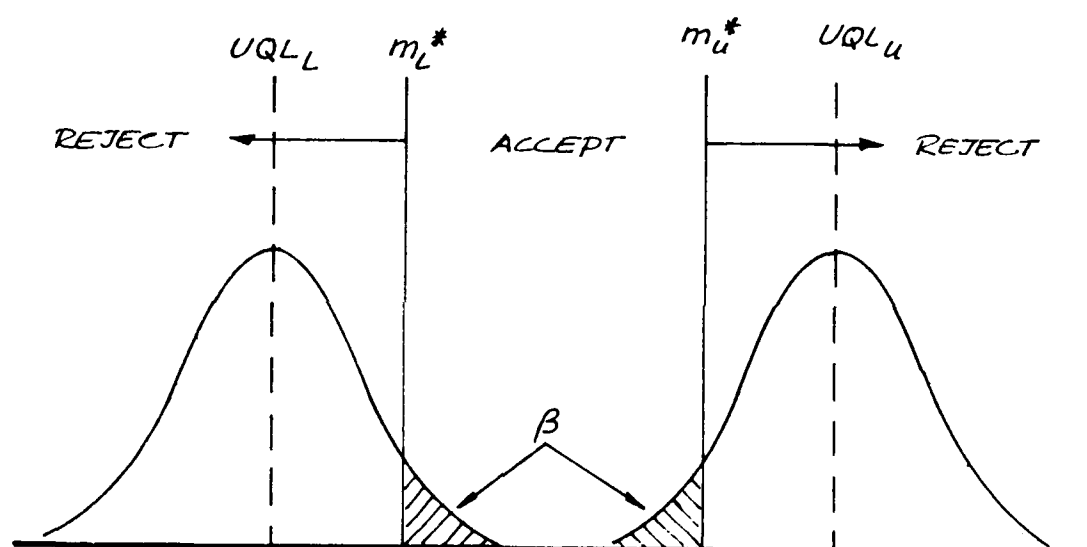
Figure 34 -- Sampling variability of two lifts of material, one with true mean equal to UQL_U, the other with true mean equal to UQL_L.
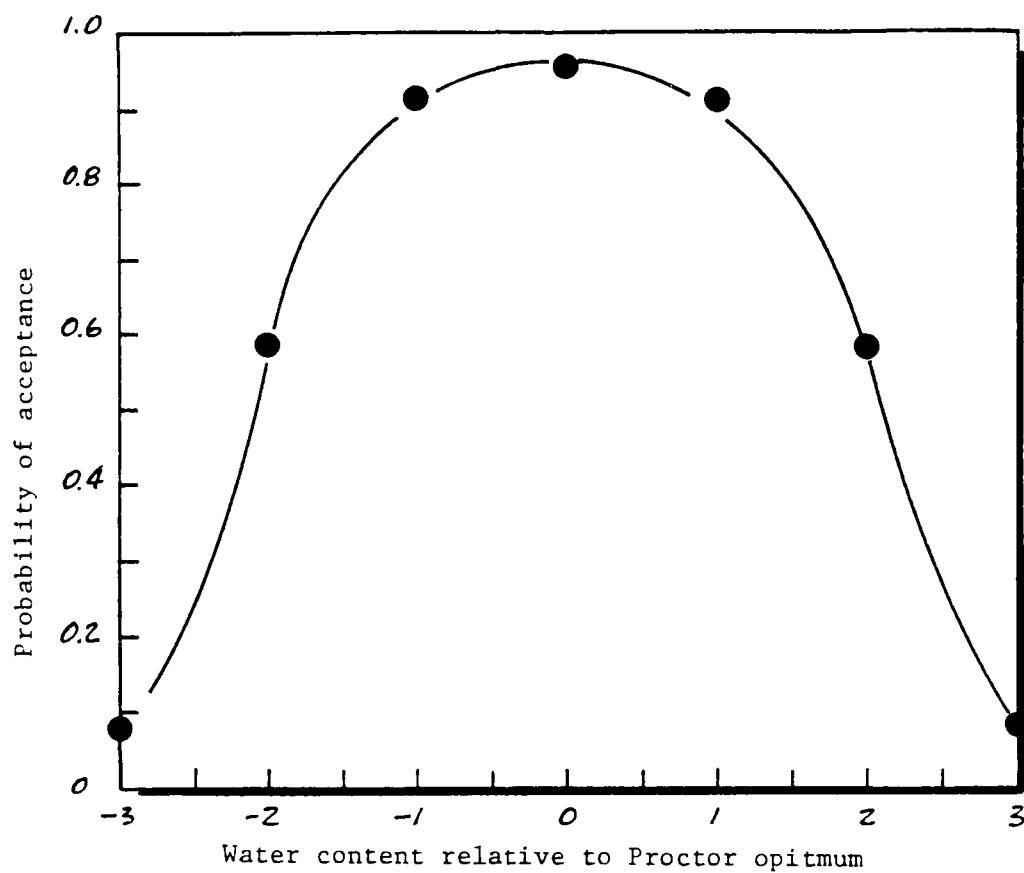
Figure 35 -- Operating characteristic curve for an acceptance sampling
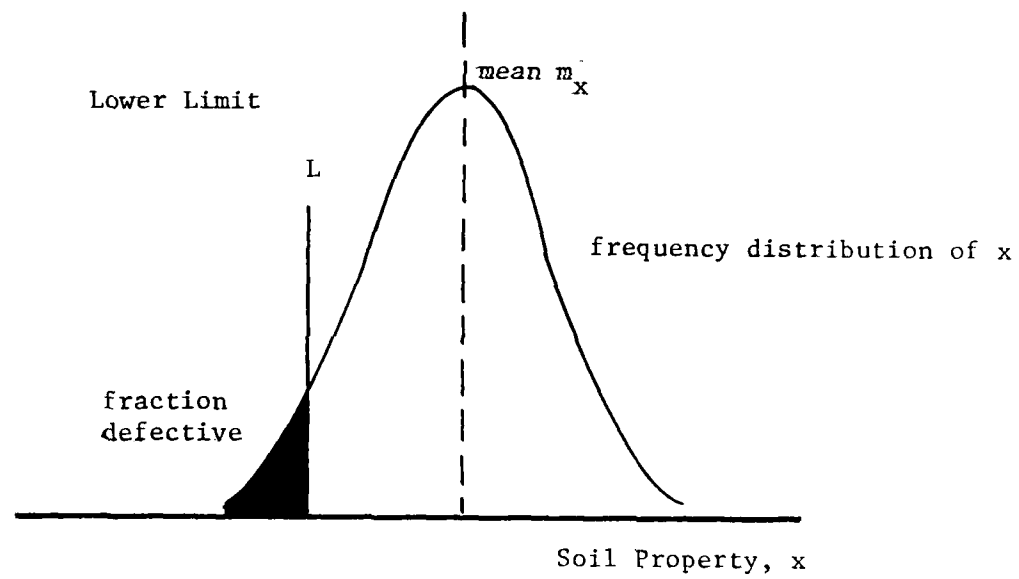plan to assure the value of the mean, with double
sampling limits.

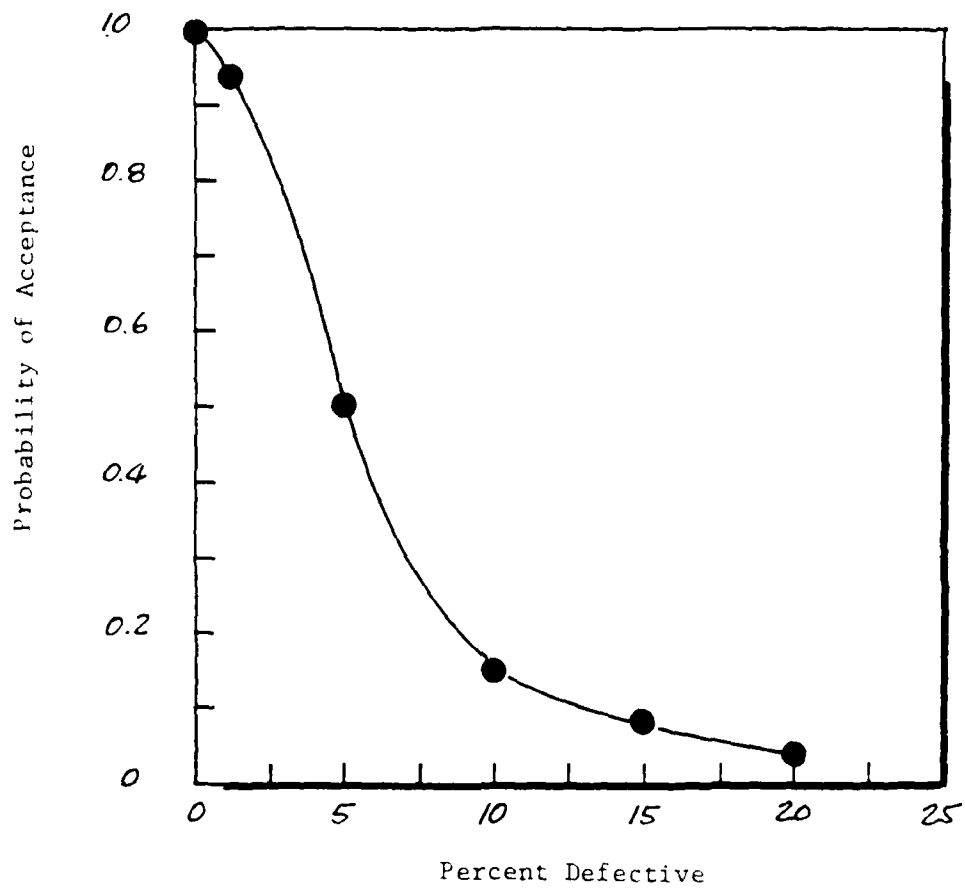Figure 36 -- Fraction defective for a Normal frequency distribution of soil properties.

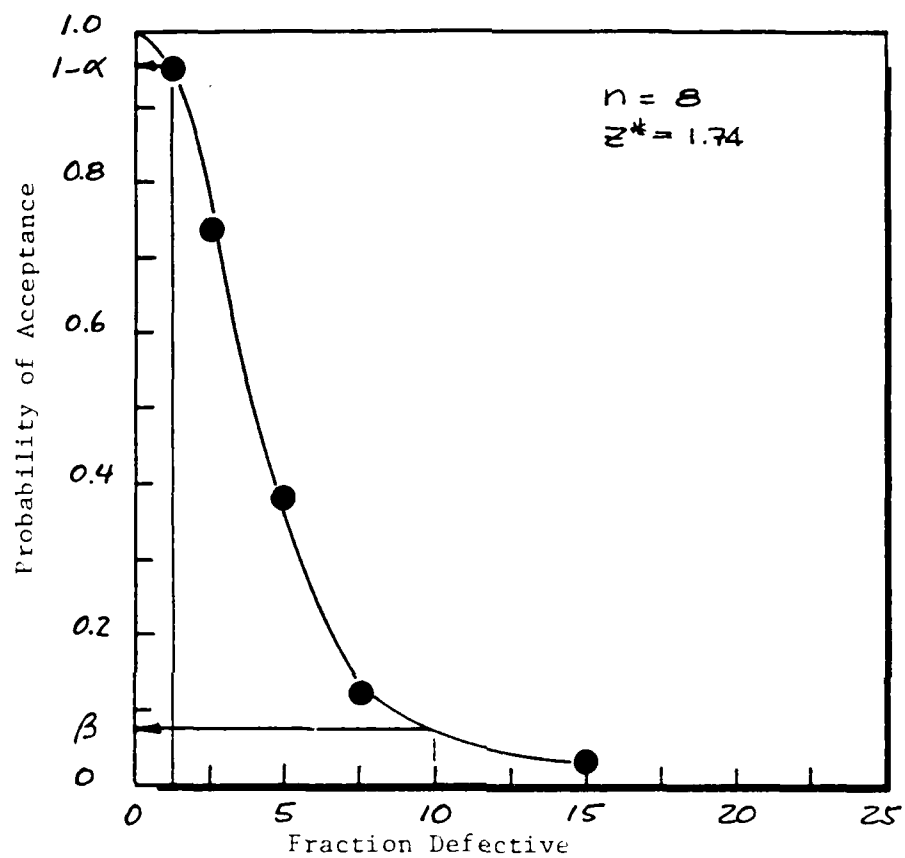Figure 37 -- Operating characteristic for fraction defective sampling with single sampling limits, standard deviation known.

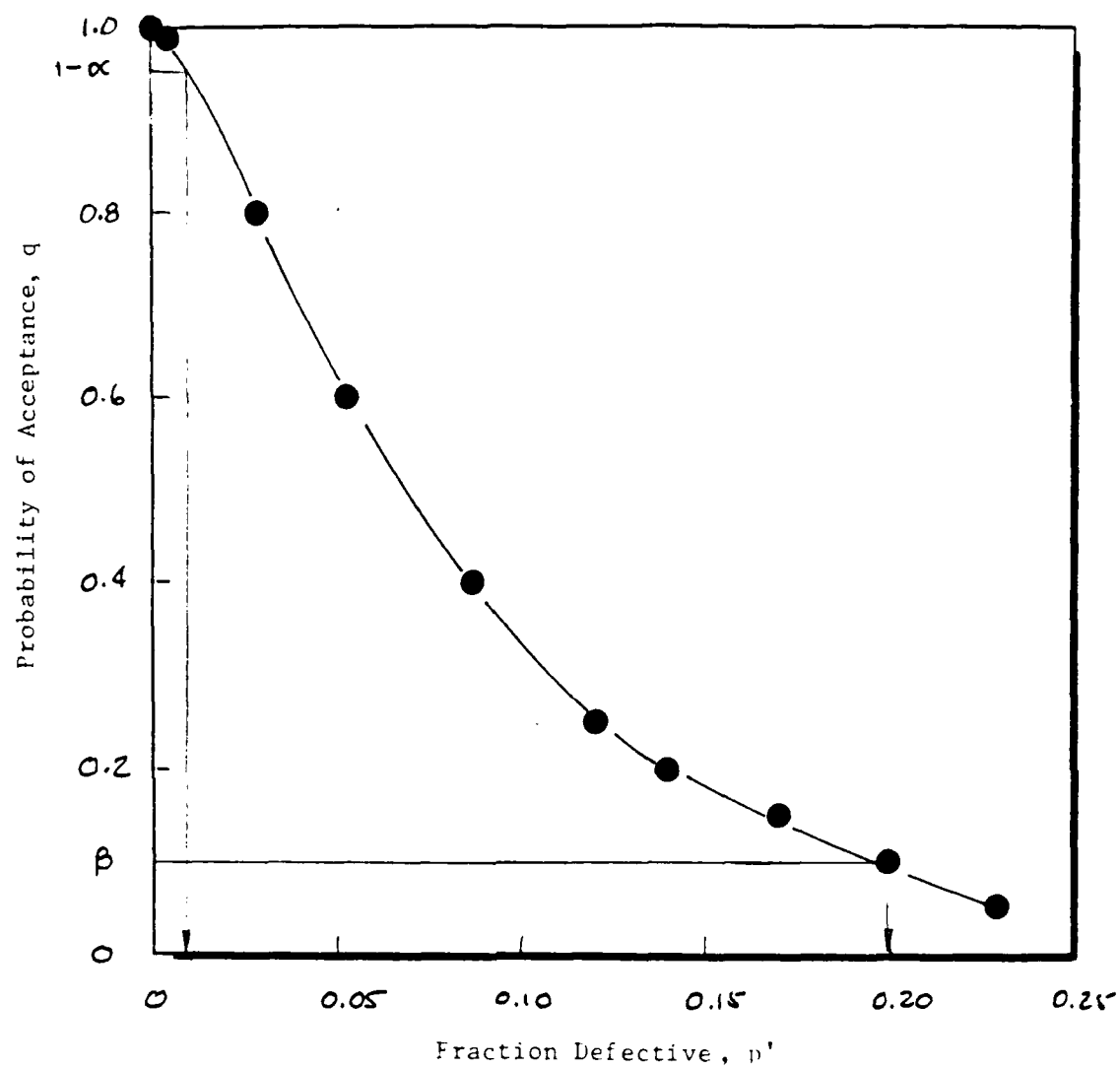Figure 38 -- Operating characteristic for acceptance sampling plan derived in Plate 7.

Figure 39 -- Approximate OC curve for single limit sampling plan with standard deviation unknown (Plate 8).
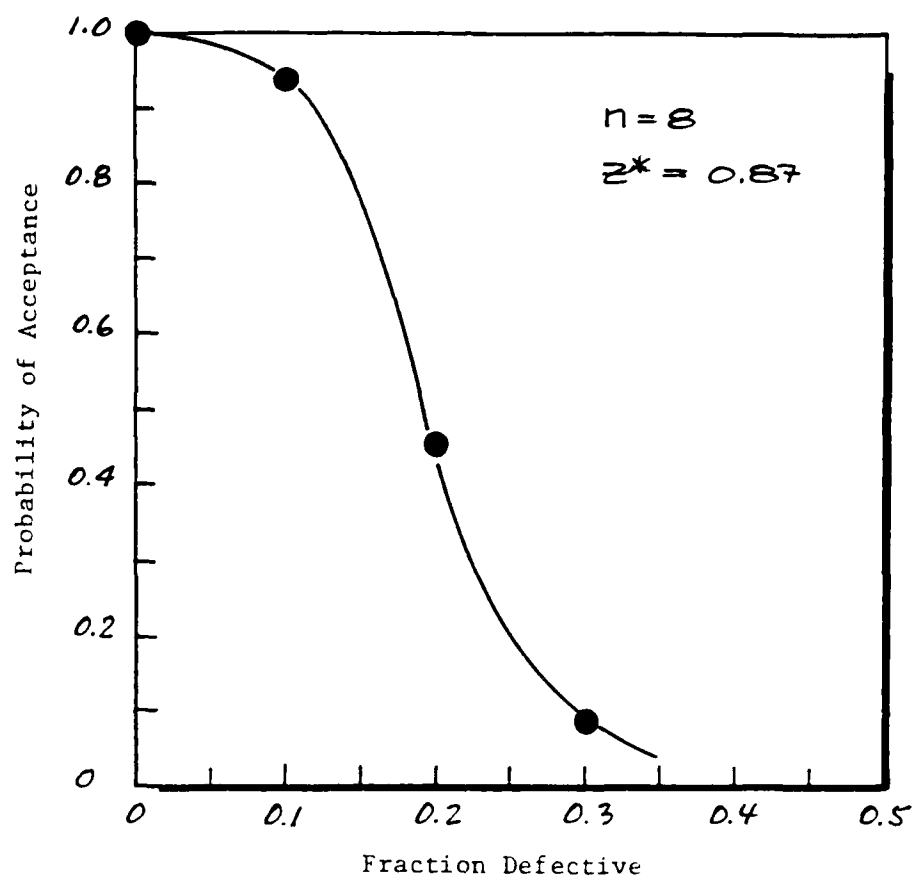
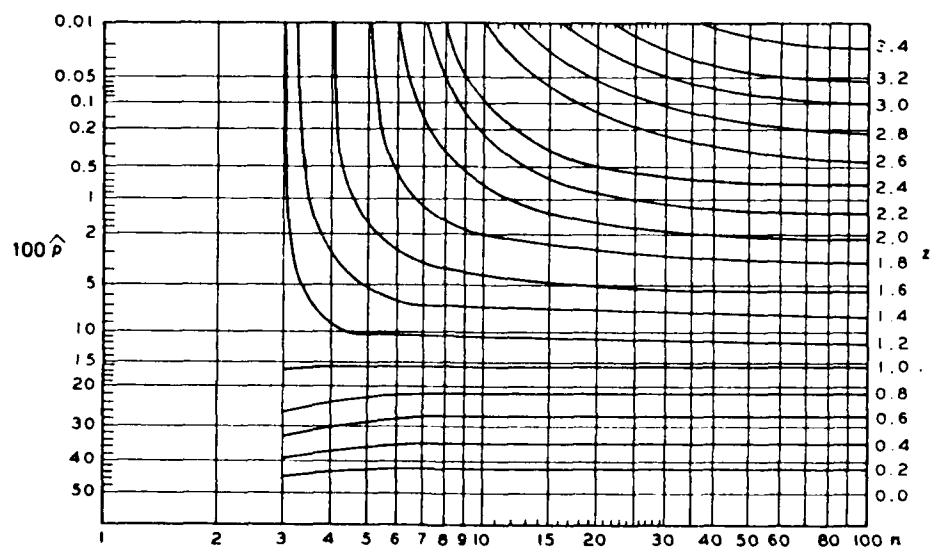Figure 40 -- Operating characteristic curve for sampling plan derived in Plate 9.
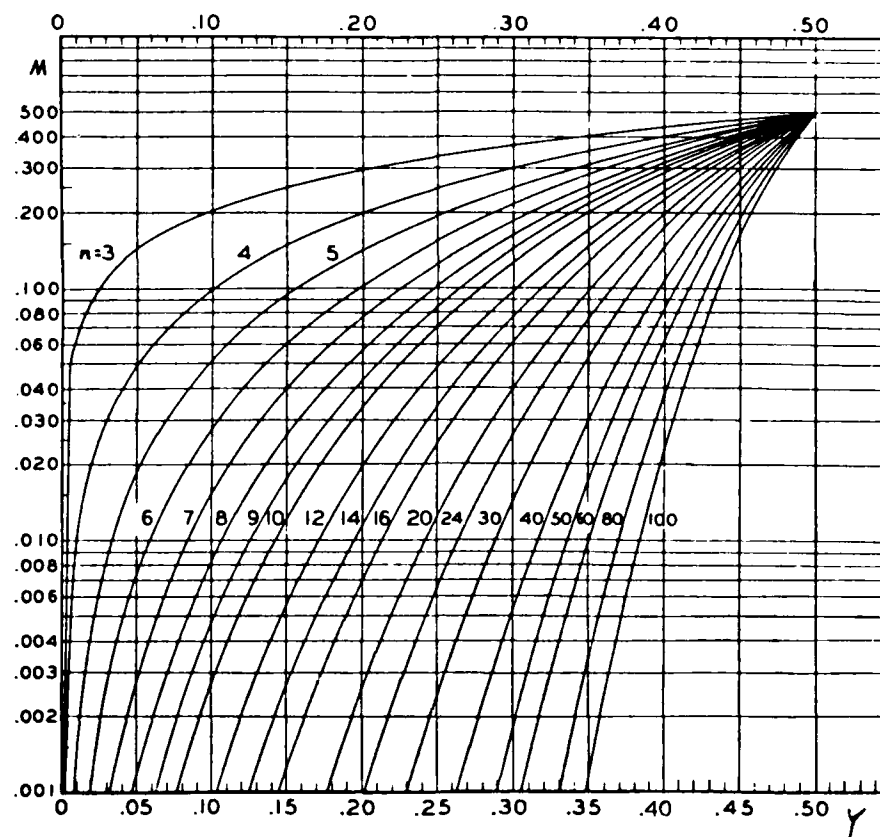
Figure 41 -- Chart for determining fraction defective from z (from Duncan, 1974).

For Standard Deviation Plans Take abscissa $= \dfrac{1 - Z\sqrt{n/(n-1)}}{2}$

For Average Range Plans Take abscissa $= \dfrac{1 - Z/\sqrt{\nu}}{2}$

Figure 42 -- Chart for determining maximum allowable fraction defective, M
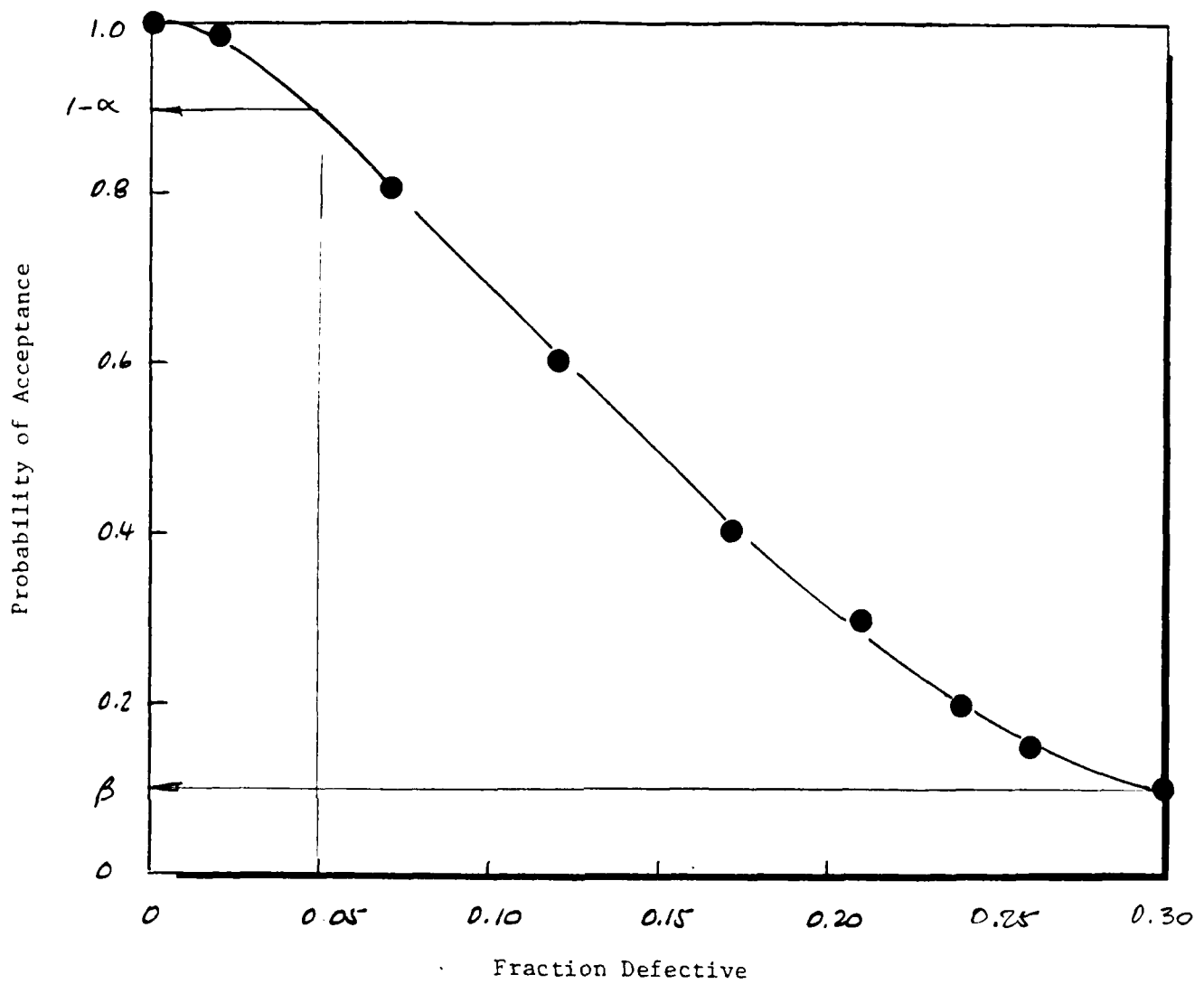(from Duncan, 1974).

Figure 43 -- Operating cahracteristic curve for sampling plan derived in Plate 10.

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